Value of the Inequality

If $m > 0$ and $n < 0$, decide if each inequality is *Always True, Sometimes True, Never True*, or *Can't Be Determined* with the given information.

Circle the correct answer.	Justify your choice.
1. $m + n < 0$ a. Always True b. Sometimes True c. Never True d. Can't Be Determined	
2. $m - n > 0$ a. Always True b. Sometimes True c. Never True d. Can't Be Determined	
3. $(m)(n) < 0$ a. Always True b. Sometimes True c. Never True d. Can't Be Determined	
4. $\frac{m}{n} > 0$ a. Always True b. Sometimes True c. Never True d. Can't Be Determined	

Teacher Notes: Value of the Inequality

Questions to Consider About the Key Mathematical Concepts

When solving problems involving operations with integers, can students generalize the results of an operation when given constraints about the numbers to be operated on? To what extent do they

- make sense of the properties of operations?
- interpret an algebraic inequality?
- use examples and counter-examples to justify their choice?

Common Core Connection (CCSS.Math.Content.7.NS.A.1; CCSS.Math.Content.7.NS.A.2; CCSS.Math.Content.7.EE.B.4)

Grade: Seventh

Domain: The Number System

Cluster:

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

Domain: Expressions and Equations

Cluster:

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Uncovering Student Understanding About the Key Concepts

Using the Value of the Inequality Probe can provide the following information about how the students are thinking about operating with integers through the interpretation of inequalities.

Do they		*Do they*
• correctly apply the rules of operating on integers?	OR	• overgeneralize rules from one operation to another?
• correctly interpret the inequalities, first in the numbers involved and second in the results of the operations?	OR	• have difficulty determining either the values of *m* and/or *n* or the value of the resulting operation?
• use properties of operations to justify a choice of “always true” and “never true”?	OR	• use only examples or counter-examples?
• use examples and counter-examples to justify a choice of “sometimes true”?	OR	• rely on only examples or only counter-examples when both are needed?

Exploring Excerpts From Educational Resources and Related Research

Common areas of difficulty for students:

Data from the Mathematics Assessment for Learning and Teaching (MaLT) project data base indicate that students demonstrate the following types of difficulties with integer operation problems: the sign and the number conceived of as two separate objects, ignoring the negative sign, viewing the division symbol as “subtract.” (Ryan & Williams, 2007, p. 218)

Relatively few students displayed understanding of integers. They tend to memorize “rules” given by their teachers. Students often struggle to remember all the different rules, and even if they could remember rules, they were often not sure which rule applied to which task. The study also found:

- Students got mixed-up with operation and signs when there are subtraction and negative signs in a problem.
- Some students’ errors are a mixture of many types of errors:
  - They take the sign of the larger number and put it in front and multiplied the sign of the numbers to get the operations.
  - They apply multiplication rule every time they saw two signs.
  - They also take the sign of the larger number and then subtract or add the rest.
- Students generally do better in multiplication and division of integers compared to addition and subtraction involving negative integers. This can be due to confusion because there are two signs of the same kind in a problem involving subtraction. It can also be due to the multiplication rule as being easier to remember. (Kahlid, Rosmah, & Badarudin, 2012, slide 15)

Surveying the Prompts and Selected Responses in the Probe

The Probe consists of four related justified list items. The prompts and selected responses are designed to elicit understandings and common difficulties as described below:

If a student chooses	*It is likely that the student*
1b, 2a, 3a, 4c (correct answers)	• correctly interpreted the values of *m* and *n*; and applied the properties of operating with integers [See Sample Student Responses 1, 2, and 3]. *Look for indication of the student's understanding in the written explanations of how the student got the answer.*
1a	• has overgeneralized the rule for multiplication and division—"a negative and a positive is always a negative" [See Sample Student Response 7].
2b	• has dropped the negative sign associated with $n < 0$ and is not interpreting the operation in terms of subtracting a negative [See Sample Student Response 4].
3b, 4b	• has overgeneralized the rule for addition—"take the sign of the larger" [See Sample Student Responses 4, 5, and 6].
Various other choices	• can apply properties for some operations but not others; or • has misinterpreted the value of *m* or *n* therefore choosing incorrectly about the value of the operation [See Sample Student Responses 5, 6, and 7].

Teaching Implications and Considerations

Ideas for eliciting more information from students about their understanding and difficulties:

- What does this inequality symbol mean? Can you give me an example of an integer that makes that inequality true?
- How do you substitute values for *m* and *n* in the inequality?
- How can you justify your choice using what you know about the properties of operating on integers rather than just providing examples?

Ideas for planning instruction in response to what you learned from the results of administering the Probe:

- Build foundational understanding of the meaning of and notations for negative numbers.
- Canceling a debt has powerful meaning behind subtracting a negative number for most students. They often relate to money and understand discussions based on it.
- Build in multiple opportunities for students to model operations with integers using chip models and number lines. Do not jump too quickly to rules such as "subtracting a negative is the same as adding a positive." Instead allow the students to form these generalizations based on modeling both processes numerous times.
- Use problem contexts and ask students to develop their own problems to represent a given numerical equation.
- Support the connection between the modeling of integer operations and the mathematical model (e.g., numerical equations).
- Help students form generalizations and represent those generalizations with algebraic equations and inequalities.

Sample Student Responses to Value of the Inequality

Responses That Suggest Understanding

Sample Student Response 1

Probe Item 1. b. If m has a greater absolute value than n, it is true. If not, it is false.

Probe Item 2. a. A positive minus a negative is always a positive.

Probe Item 3. a. A positive times a negative is always a negative.

Probe Item 4. c. A positive divided by a negative equals a negative.

Sample Student Response 2

Probe Item 1. b. A positive plus a negative takes the positive number down, but not always below zero. Ex: $2 - 4 < 0$ would be true, $4 - 2 < 0$ would not be true.

Sample Student Response 3:

Probe Item 1. b. It will totally depend on which one is farther away from zero.

Responses That Suggest Difficulty

Sample Student Response 4

Probe Item 2. b. This could be either positive or negative, depending on which numbers are chosen for m and n.

Probe Item 4. b. This also depends on the numbers chosen . . . it could be either positive or negative.

Sample Student Response 5

Probe Item 1. c. If something larger than zero is added to something smaller than zero the result will still be larger than zero, not smaller.

Probe Item 3. d. This can't be determined without knowing what m or n are.

Sample Student Response 6

Probe Item 1. c. If $m > 0$, then $m + n$ isn't possibly less than 0.

Sample Student Response 7

Probe Item 1. a. When adding a number that is greater than zero to a number that is not, the answer is always less than zero.

Probe Item 2. b. When subtracting a number that is greater than zero from one that is not, the answer is almost always positive.

Sample Student Response 8

Probe Item 1. a. If $m = 1$ and $n = -20$ then yes.

Probe Item 2. b. If $m = 1$ and $n = -5$, then yes.