
A User's Guide to Learning Styles and Math Tools

The journey of developing this book began almost five years ago. After over sixty combined years of service in schools, two of us—John R. Brunsting and Terry Walsh—were coming to the end of our careers as mathematics instructors and administrators. For most of those sixty years, we had the pleasure of working together in Hinsdale Central High School in Hinsdale, Illinois, where we met Harvey Silver and were introduced to the *Thoughtful Classroom* professional development model he designed with Richard Strong. What we quickly came to learn is that the *Thoughtful Classroom* really works. Whenever we implemented *Thoughtful Classroom* strategies in our classrooms or worked with other teachers to help them implement *Thoughtful Classroom* strategies in their own classrooms, the effect on student learning was palpable—students became more engaged, discussions got richer, student thinking went deeper, and test scores went up.

There was, however, one particular *Thoughtful Classroom* text that always seemed to make the biggest difference in classrooms in the shortest amount of time. That text was *Tools for Promoting Active, In-Depth Learning* (Silver, Strong, & Perini, 2001; Silver, Strong, & Commander, 1998). The idea behind *Tools for Promoting Active, In-Depth Learning* is simple. It is a collection of classroom-tested tools, or simple teaching “moves,” that teachers can use to foster active, in-depth learning. These tools are based on the principles of effective learning and brain-based instruction and require little or no planning. As such, the tools can serve as “on-the-fly” techniques whenever a learning episode begins to lag, or they can be planned into a lesson or unit ahead of time in order to meet specific objectives.

As we—John and Terry—ended our careers in classrooms and began new ones as staff developers, we began exploring a new question with Harvey: What would a tools-based approach to mathematics instruction look like? Or, more specifically, rather than adapting generic tools to the demands of mathematics instruction, why not develop a new book, one that would respond directly to the concerns of mathematics teachers? The idea of a math-specific tools book excited all three of us, so we set out to align what we were doing in mathematics staff development with this new “math tools” venture. We worked with teachers of mathematics from around the country to select tools that were most relevant to mathematics instruction. We modified other tools to make them more math-centered, and we developed some new ones along the way.

As we added tools to our list, we organized them into four distinct styles of instruction: a *Mastery* style that emphasizes skill acquisition and retention of critical mathematical terms; an *Understanding* style that builds students’ capacities to find patterns, reason and prove, and explain mathematical concepts; a *Self-Expressive* style that capitalizes on students’ powers of imagination and creativity; and an *Interpersonal* style that invites students to find personal meaning in mathematics by working together as part of a community of problem-solvers. However, the more we discussed our aim to develop a book that would provide teachers of mathematics with a repertoire of instructional tools that they could use to differentiate instruction according to different styles, the more we heard a common refrain. It sounded like this: We know that different students have different ways of learning and that we need to engage them all, but isn’t mathematics a worst-case scenario for differentiation?

It isn’t difficult to see why mathematics might seem like a worst-case scenario for differentiated instruction. The quantitative nature and sequential organization of mathematical content can make considerations of student differences seem marginal. Add to this the realities of teaching in an age marked by standards, high-stakes tests, and teacher accountability laws—an age when you can pick up almost any mathematics journal and find a piece that sounds the alarm for all-out reform, a piece that sounds like this:

Efforts to improve the quality of mathematics education in the United States have been under way for the past half-century. According to the 2007 National Assessment of Educational Progress (NAEP), however, more than half of our fourth graders and almost 70% of our eighth graders still fail to achieve proficiency in mathematics. Our students also continue to fare poorly on international assessments of mathematics achievement. On the 2003 Program for International Student Assessment (PISA) exam, which tests students’ mathematical literacy and problem-solving abilities, students from 23 out of the 39 participating countries significantly outperformed students from the United States. Even our top students lagged behind their counterparts from other nations. Despite decades of reform, then, it is clear that we still have a serious mathematics problem in the United States. Because we live in a world where individuals who possess well-developed mathematical skills are more likely to go to college, more likely to be employed, and more likely to earn higher salaries than those who do not, it is even clearer that we must find a solution.

Clearly, the refrain of mathematics teachers about the difficulties in differentiating instruction had some real wisdom behind it. What they were asking was: Can I *afford* to differentiate? With the stakes so high, how much attention can I really pay to the differences among my students? After all, the standards I'm being asked to meet aren't differentiated; they're uniform, the same for every single student regardless of style or ability level.

What we've discovered during our journey as teachers of mathematics, administrators, professional developers, and authors is that *differentiation is not in the way of meeting high standards, it is the key to meeting them*. Our students' perceptions of mathematics as a discipline, their academic success in our classrooms, and their development as math-literate citizens all depend on our ability to engage all our students, not just our "math whizzes" and high achievers. And no, mathematics is not a worst-case scenario for differentiation. Mathematics can be differentiated as easily as language arts or social studies or any other subject for that matter. All you need to understand are two little words: *learning styles*.

WHAT ARE LEARNING STYLES AND WHY DO THEY MATTER?

Few ideas in education have stood the test of time as well as learning styles. The history of style stretches all the way back to the work of Carl Jung (1923), one of the founding fathers of modern psychology. What Jung discovered is that the ways in which people process and evaluate information tend to develop into particular personality types. Years later, Kathleen Briggs and Isabel Myers (1962/1998) took Jung's work and expanded on it to create a comprehensive model of cognitive diversity. The fruit of Briggs and Myers' efforts is the world-renowned Myers-Briggs Type Indicator, which, according to recent estimates, some two million people take each year to better understand their strengths and liabilities as learners, workers, and individuals. In the years since the development of the Myers-Briggs Type Indicator, new generations of educational researchers including Bernice McCarthy (1982), Carolyn Mamchur (1996), Edward Pajak (2003), Gayle Gregory (2005), and Harvey F. Silver, Richard Strong, and Matthew Perini (2007) have adapted and refined these ideas and helped educators across the globe put learning styles to work in classrooms and schools.

In a development of special interest to teachers of mathematics, Harvey F. Silver, Ed Thomas, and Matthew Perini (2003) applied the research on learning styles specifically to the study of mathematics. Out of their work came the identification of four distinct mathematical learning styles, outlined in Figure 1.1.

It goes without saying that no student falls completely into one style category. Learning styles should never be used to reduce students to a set of identifiable behaviors neatly summarized in a three-inch by three-inch box. However, most of us tend to develop clear preferences for certain styles, while seeking to avoid other styles.

To get a better sense of what the four mathematical learning styles look like in the classroom and to help you discover which styles you prefer, let's look in on the classrooms of four different teachers of mathematics. While students in each of these four mathematics classrooms are all studying area and perimeter, each teacher is

Figure 1.1 The Four Types of Mathematics Students

The Four Types of Mathematics Students	
<p style="text-align: center;">Mastery Math Students . . .</p> <p><i>Want to . . .</i> learn practical information and set procedures.</p> <p><i>Like math problems that . . .</i> are like problems they have solved before and that use algorithms to produce a single solution.</p> <p><i>Approach problem solving . . .</i> in a step-by-step manner.</p> <p><i>Experience difficulty when . . .</i> math becomes too abstract or when faced with non-routine problems.</p> <p><i>Want a math teacher who . . .</i> models new skills, allows time for practice, and builds in feedback and coaching sessions.</p>	<p style="text-align: center;">Interpersonal Math Students . . .</p> <p><i>Want to . . .</i> learn math through dialogue, collaboration, and cooperative learning.</p> <p><i>Like math problems that . . .</i> focus on real-world applications and on how math helps people.</p> <p><i>Approach problem solving . . .</i> as an open discussion among a community of problem solvers.</p> <p><i>Experience difficulty when . . .</i> instruction focuses on independent seatwork or when what they are learning seems to lack real-world application.</p> <p><i>Want a math teacher who . . .</i> pays attention to their successes and struggles in math.</p>
<p style="text-align: center;">Understanding Math Students . . .</p> <p><i>Want to . . .</i> understand why the math they learn works.</p> <p><i>Like math problems that . . .</i> ask them to explain, prove, or take a position.</p> <p><i>Approach problem solving . . .</i> by looking for patterns and identifying hidden questions.</p> <p><i>Experience difficulty when . . .</i> there is a focus on the social environment of the classroom (e.g., on collaboration and cooperative problem solving).</p> <p><i>Want a math teacher who . . .</i> challenges them to think and who lets them explain their thinking.</p>	<p style="text-align: center;">Self-Expressive Math Students . . .</p> <p><i>Want to . . .</i> use their imagination to explore mathematical ideas.</p> <p><i>Like math problems that . . .</i> are non-routine, project-like in nature, and that allow them to think “outside the box.”</p> <p><i>Approach problem solving . . .</i> by visualizing the problem, generating possible solutions, and exploring among the alternatives.</p> <p><i>Experience difficulty when . . .</i> math instruction is focused on drill and practice and rote problem solving.</p> <p><i>Want a math teacher who . . .</i> invites imagination and creative problem solving into the math classroom.</p>

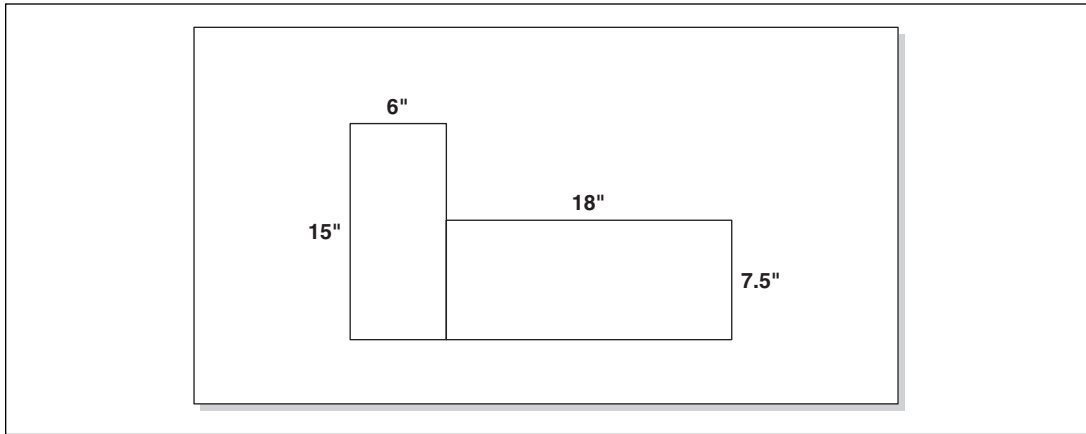
SOURCE: Silver, Thomas, Perini (2003).

approaching the content in a different way. William Merkel, Sandy Horowitz, Bruce Wong, and Julia Lacomba all teach sixth-grade mathematics, and each teacher has developed a different activity for students to complete. Which of these classrooms would you want to be in the most? Which of these classrooms would you want to be in the least? Once you are familiar with the four classroom activities, rank them in order of preference from most preferred to least preferred.

Mastery Activity

In William Merkel's classroom, students have just reviewed the formula for finding the area and perimeter of a rectangle. William wants to assess his students' progress to see if they have mastered the procedure. He provides each student with the drawing of an irregular shape and explains, "We have gone over how to find the area and perimeter of a rectangle. Today we are going to look at an irregular shape. I want you to apply the formulas you already know about area and perimeter to compute the area and perimeter of this irregular shape."

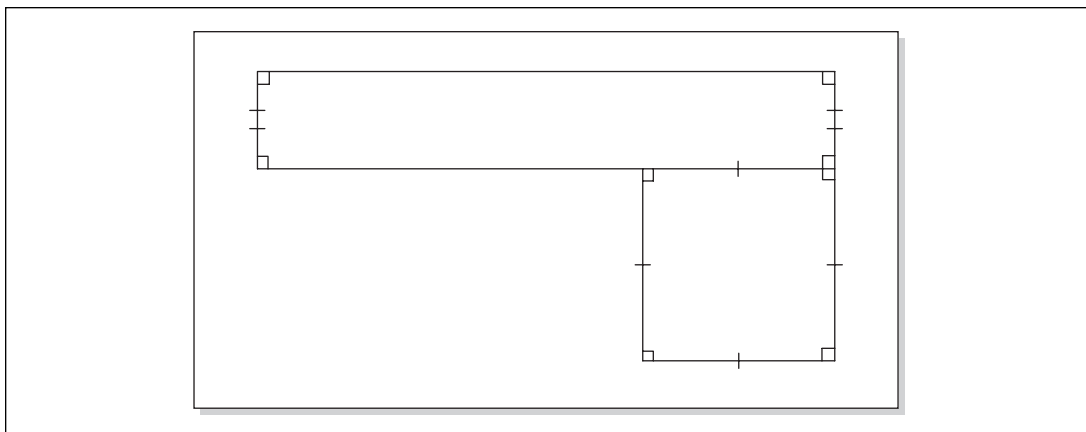
Figure 1.2 Mathematics Classroom—Mastery Activity



Understanding Activity

Sandy Horowitz wants to see if her students understand what measurements are needed to correctly calculate area and perimeter. Her students are familiar with the procedures for finding area and perimeter, and today she is also challenging students with an irregular shape. Sandy starts by providing her students with a diagram without any measurements. She continues, "I want you to figure out what would be the fewest measurements needed to accurately calculate both the area and perimeter of this irregular shape. Then, I want you to explain the process you used to figure out your answer."

Figure 1.3 Mathematics Classroom—Understanding Activity



Self-Expressive Activity

After studying area and perimeter with his class, Bruce Wong wants to inspire his students to think more divergently about mathematics. Today, Bruce’s class will be working on an open-ended assignment. Bruce tells his students that they will be working with four shapes—one rectangle, one square, one trapezoid, and one equilateral triangle. He elaborates on the assignment, “I want each of you to create your own area and perimeter problem by connecting these four shapes. The shapes can be of any measurement you choose. You can arrange the shapes in any way you want, but you have to be able to solve your problem using only four measurements.” For his students that finish the assignment early, Bruce challenges them to create another problem connecting the same four shapes in a different way. However, this time students must be able to solve the problems they create using only three, two, or even one measurement.

Interpersonal Activity

Julia Lacomba always tries to have her students find personal relevance in mathematics, and the topic of area and perimeter is no different. Today, Julia is asking her students to draw up a floor plan of their homes, illustrating the dimensions of each room. “I want each of you to picture your home. It doesn’t matter how big or small it is, or whether you live in an apartment or a house. Think about each room in your home. What are the dimensions of each room? How do the rooms connect to each other? Draw a floor plan for your home that includes the dimensions of each room.” After students have finished their floor plans with estimated measurements, Julia challenges students to answer these two questions: “Suppose that you wanted to carpet each room of your home (except the bathroom and kitchen). How much carpet would you need? What if you wanted to install crown molding or put up a new wallpaper border in each of these rooms? How much total molding or border would you need?”

Each of the four teachers we have just met wants his or her students to learn and understand what area and perimeter are, and how to calculate area and perimeter in different ways. However, each of these teachers has developed a very different activity to use. So, which classroom activity would you enjoy most? Which would you like least or try to avoid? If you have a preference for one activity over the others, then this is the first signal of your mathematical learning style. Your style influences the types of activities you enjoy and how you approach learning and teaching.

If you enjoyed William Merkel’s activity, then your dominant style preference is most likely *Mastery*. Students who prefer the Mastery style learn best step-by-step and enjoy activities with clear procedures and one correct answer.

If you selected Sandy Horowitz’s activity, then your dominant style preference is most likely *Understanding*. Students who prefer the Understanding style enjoy analytical tasks where they have to figure things out and explain or prove their answers.

If you were drawn to Bruce Wong’s activity, then your dominant style preference is most likely *Self-Expressive*. Students who prefer the Self-Expressive style thrive when they are given choices, have the opportunity to be creative, or are asked to explore alternative solutions to problems.

If you chose Julia Lacomba’s activity, then your dominant style preference is most likely *Interpersonal*. Students who prefer the Interpersonal style learn best from others or when the content has a strong relevance to their lives. These students do well with activities that are personal, connected to their lives, or that result in helping others.

Now that you have a deeper understanding of how the four styles play out in mathematics instruction, ask yourself this: What happens if we combine the work of William, Sandy, Bruce, and Julia? What if over the course of the unit on area and perimeter, students completed all four of these activities? We get a differentiated mathematics classroom, one in which every learner gets what he or she wants and needs, while also growing underdeveloped capacities by working in weaker styles. In short, we get an instructional program that asks students to:

- Apply formulas, compute accurately, and reinforce skills through practice (Mastery).
- Discover patterns, make generalizations, and develop mathematically sound explanations (Understanding).
- Think creatively, develop new problems, and try out a variety of problem-solving approaches (Self-Expressive).
- Make personal connections and solve real-world math problems (Interpersonal).

Thus, style-based mathematics instruction is more than a way to invite a greater number of students into the teaching and learning process. Style-based instruction is, plain and simple, good math—balanced, rigorous, and diverse. Fifth-grade teacher Barb Heinzman puts it this way:

What I saw right away was that not only did different students approach mathematics using different learning styles, but real mathematical power required using all four styles. Think about it: If you can't compute accurately, explain your ideas, discover solutions, and apply math in the real world—you don't know math. Miss even one of these and you miss the boat. The problem with most math programs is they emphasize just one of these and leave out the rest. By building every unit so it includes all four styles of learning, I support all my students, and I stretch them into areas where they wouldn't naturally go. (Strong, Silver, & Perini, 2001, p. 79)

Barb's experiences as a teacher of mathematics are borne out by current research showing that style-based teaching leads to improved learning and higher levels of achievement. For example, Robert J. Sternberg (2006) and his colleagues conducted a remarkable series of studies involving diverse student populations including students from Alaskan Eskimo villages, rural Kenya, and a wide range of student populations from across the United States. In these studies, students were taught mathematics (along with other subjects) in five different ways:

- A memory-based approach emphasizing identification and recall of facts and concepts;
- An analytical approach emphasizing critical thinking, evaluation, and comparative analysis;
- A creative approach emphasizing imagination and invention;
- A practical approach emphasizing the application of concepts to real-world contexts and situations; and
- A diverse approach that incorporated all the approaches.

Out of these studies, Sternberg and his colleagues drew two conclusions. First, whenever students were taught in a way that matched their own style preferences,

those students outperformed students who were mismatched. Second, and even more important, students who were taught using a diversity of approaches outperformed all other students on both performance assessments and on multiple-choice memory tests. Sternberg (2006) goes on to say, “In other words, even if our goal is just to maximize students’ retention of information, teaching for diverse styles of learning still produces superior results. This approach apparently enables students to capitalize on their strengths and to correct or to compensate for their weaknesses, encoding material in a variety of interesting ways” (pp. 33–34).

So now that we know what learning styles are, what they look like when applied in the mathematics classroom, and that teaching with learning styles in mind leads to improved teaching and higher levels of learning, how do we incorporate them into our own classrooms? By using a variety of tools. Chapters 2 through 5 of this book each contain a set of tools that support one of the four mathematical learning styles. Mastery tools (Chapter 2) increase retention of critical terms and deepen students’ computation and practice skills. Understanding tools (Chapter 3) challenge students to take an analytical approach to mathematics. Self-Expressive tools (Chapter 4) allow students to use their creativity and imagination to explore mathematical ideas. Interpersonal tools (Chapter 5) draw out the personal and social aspects of mathematics.

HOW DO I SELECT THE RIGHT TOOL FOR THE RIGHT LEARNING SITUATION?

A quick glance at the table of contents shows that the four mathematical learning styles serve as the framework for this book. Chapters 2 through 5 contain tools that support Mastery, Understanding, Self-Expressive, and Interpersonal styles, respectively. Chapter 6 provides four different strategies for combining tools from all four styles to design more powerful tests, lessons, assessment systems, and units of study. Our intent in designing the text around styles is to make the important goal of differentiating mathematics instruction eminently manageable for teachers. By selecting tools from different chapters of the book, teachers naturally accommodate and challenge learners of all four styles.

However, helping teachers of mathematics diversify their teaching practices has not been our sole purpose in writing this book. More generally, *Math Tools* has been written to provide all mathematics teachers with a repertoire of high-impact instructional techniques that they can begin using tomorrow, techniques that:

- Help students meet rigorous academic standards.
- Are backed by a reliable research base.
- Can be used to meet a host of instructional objectives, from preparing students for new learning all the way to developing performance-based assessments that ask students to show what they have learned.

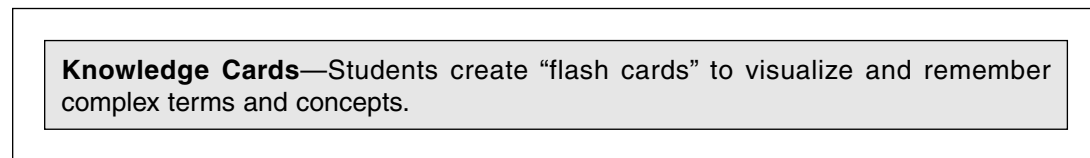
That’s why we begin Chapters 2 through 5 with a Math Tools Matrix. Each Math Tools Matrix lays out the tools within the chapter and provides the reader with an at-a-glance overview of each tool. Take a look at the Math Tools Matrix for Chapter 2 (pages 18–19). Notice how the tools are listed and described down the left side of the matrix. If you follow the top row across the two pages, from left to right, you’ll also notice that the columns are broken up into three distinct sections labeled

“NCTM Process Standards,” “Educational Research Base,” and “Instructional Objectives” (also known as “The Seven P’s”). By tracking a tool across the matrix, you can gather the vital statistics for that tool to help you determine how well it fits your purposes. To see how this process works, let’s use the first tool in the book, Knowledge Cards, to take a quick tour of a Math Tools Matrix.

Vital Statistic 1: Title and Flash Summary

The name of a particular tool is often not enough to give new readers a sense of how a tool works or to jog the memories of readers who are using *Math Tools* more like a reference text. So, after the page number that tells you where to find the tool, the first thing you’ll see is the tool’s name and a “flash summary” that describes it in one sentence or less. Figure 1.4 below shows this title and flash summary cell for the Knowledge Cards tool.

Figure 1.4 Math Tools Matrix—Knowledge Cards Title and Summary



Vital Statistic 2: NCTM Process Standards

In developing ten comprehensive standards for mathematics instruction, the NCTM has provided all mathematics instructors with a map of the terrain—an overarching set of goals to drive decisions about planning, teaching, assessment, and curriculum design. The first five of these standards address specific areas of mathematical content. You will not find these NCTM Content Standards on the Math Tools Matrix. Why? Because tools are not content-specific; they can be used to deliver instruction in any content—from addition facts to the use of geometric principles in Renaissance art.

But while tools are not content-specific, they are thinking-specific. That is to say, each tool engages students in one or more mathematical thinking processes. That’s where the back half of the NCTM Standards, known as the Process Standards, come into the picture (National Council of Teachers of Mathematics, 2000). These five standards help teachers keep teaching and learning focused on the development of the following key mathematical thinking processes.

1. *Problem Solving*, or building students’ capacity to analyze problems, develop and implement problem-solving strategies, and evaluate the effectiveness of their solutions.

2. *Reasoning and Proof*, or developing students’ ability to support claims mathematically, explain how the mathematics they learn works, and justify the choices they make as problem solvers.

3. *Communication*, or helping students clarify and deepen their thinking through listening, reading, writing, and exchanging ideas with fellow learners and problem solvers.

4. *Connections*, or expanding students' opportunities to explore the deep relationships between mathematical concepts as well as how mathematics is used in the world beyond school.

5. *Representation*, or helping students explore and gain proficiency in the different ways in which mathematical ideas can be expressed and translated into alternate forms.

If you follow the Knowledge Cards row across the page (or look at Figure 1.5), you will see that the Communication and Representation fields contain darkened circles, while the other three Process Standards have empty fields. This tells you that using Knowledge Cards will help you and your students meet the Communication and Representation standards.

Figure 1.5 Math Tools Matrix—NCTM Process Standards for *Knowledge Cards*

MASTERY				
NCTM PROCESS STANDARDS				
Problem Solving	Reasoning and Proof	Communication	Connections	Representation
		●		●

Vital Statistic 3: Educational Research Base

When it comes to instruction, we know better than ever before which techniques and strategies work. Numerous “meta-analytic” studies—studies that combine the results from many other research studies to create a larger and more reliable field of data—have helped the educational community to identify a set of best practices that consistently yield results in the classroom. Of these meta-analytic studies, there is one in particular that stands out: Robert Marzano, Debra Pickering, and Jane Pollock’s *Classroom Instruction That Works: Research-Based Strategies for Increasing Student Achievement* (2001). By comparing the effects of different instructional strategies on student performance, the researchers at Mid-continent Research for Education and Learning (McREL) identified and ranked the nine classroom practices that lead to the greatest gains in student achievement (Marzano, Pickering, & Pollock, 2001).

1. *Identifying similarities and differences*: Comparisons, analogies, metaphors, and classification strategies.

2. *Summarizing and note taking*: Teaching students how to collect, record, and condense information.
3. *Reinforcing effort and providing recognition*: Developing a positive classroom environment in which student work and achievement are a significant part of the classroom conversation.
4. *Homework and practice*: Strategies that allow students to rehearse and retain their learning both in the classroom and on their own.
5. *Nonlinguistic representation*: Using visualization, icons, symbols, and graphic organizers to represent learning.
6. *Cooperative learning*: Creating structures that allow students to work, learn, and develop products and performances as part of productive teams.
7. *Setting objectives and providing feedback*: Helping students identify goals, monitor progress, and develop plans for improvement.
8. *Generating and testing hypotheses*: Developing students' abilities to infer, interpret, and explain through inquiry and investigation.
9. *Cues, questions, and advance organizers*: Helping students activate prior knowledge, connect to new learning, and see the "structure" of what they're about to learn.

Every tool in this book has been chosen or designed with this essential research in mind. In making this strong connection to Marzano, Pickering, and Pollock's research, we have attempted to give teachers of mathematics an easy way to plan and implement research-based lessons. Whenever you select a tool for use in your classroom, you can be sure that it has a reliable research base behind it. Plus, this connection to *Classroom Instruction That Works* gives you a simple way to document how your lesson plans incorporate current and widely respected research.

If you follow our left-to-right tour across the Math Tools Matrix on pages 18–19, you'll see that to Marzano, Pickering, and Pollock's nine categories, we have added two more: *Vocabulary* and *Writing*. Here's why:

10. *Vocabulary*: A large number of studies show that direct vocabulary instruction focused on the most critical academic terms (as opposed to long lists of unprioritized words) yields significant improvement in student comprehension and achievement. In fact, Marzano (2004) shows that effective vocabulary instruction can increase student comprehension by as much as 33 percentile points. Put another way, a student whose understanding of content puts him at the 50th percentile without any vocabulary instruction can move all the way to the 83rd percentile if his teacher provides direct instruction in essential academic terms.
11. *Writing*: "Writing," Douglas Reeves (2002) tells us, "improves performance in all academic areas" (p. 5). And it's not hard to see why. When students are given meaningful opportunities to process ideas in writing, to stop the flow of content and summarize, or elaborate on, or explore connections to other disciplines or their own lives, or develop their own imaginative responses to new content (mathematical or otherwise), the depth of their understanding increases dramatically.

Figure 1.6 below shows this second set of columns labeled “Educational Research Base” on the Math Tools Matrix. The Knowledge Cards tool, with its emphasis on sketching and summarizing critical math terms incorporates three different research-based practices: summarizing and note taking, nonlinguistic representation, and direct vocabulary instruction.

Figure 1.6 Math Tools Matrix—Educational Research Base for *Knowledge Cards*

EDUCATIONAL RESEARCH BASE										
Identifying similarities and differences	Summarizing and note taking	Reinforcing effort and providing recognition	Homework and practice	Nonlinguistic representations	Cooperative learning	Setting objectives and providing feedback	Generating and testing hypotheses	Questions, cues, and advance organizers	Vocabulary	Writing
	●			●					●	

Vital Statistic 4: Instructional Objectives

Any time we select a tool to use in our classroom or incorporate into our lesson or unit designs, we are seeking a way to meet specific instructional objectives or questions. For example:

- How will I **p**repare students for new learning?
- How will I **p**resent new content in a way that is engaging?
- How will students **p**ractice new skills effectively?
- How will students **p**rocess new content deeply?
- How will I engage students in meaningful **p**roblem solving?
- How will students demonstrate or **p**erform what they know and understand?
- How will students **p**ersonalize their learning so that it is meaningful to them?

This list of questions is called the Seven P’s (for more on how to use the Seven P’s to design comprehensive lessons and units, see pages 253–256). The Seven P’s are based on our synthesis of the work of a number of educational researchers concerned with lesson and unit design, including Madeline Hunter (1984), Grant Wiggins and Jay McTighe (2005), and Robert Marzano (2003). By adapting this work to fit the specific demands of the mathematics classroom, the Seven P’s serve as a simple framework for matching tools to your own classroom objectives.

So now let’s complete our tour of the Math Tools Matrix. If we follow the Knowledge Cards row all the way down to the last set of columns labeled “Instructional Objectives,”

which contain The Seven P's, we see two different P's represented: *Processing* and *Personalizing* (see Figure 1.7 below).

Figure 1.7 Math Tools Matrix—Instructional Objectives for *Knowledge Cards*

INSTRUCTIONAL OBJECTIVES						
Preparing	Presenting	Practicing	Processing	Problem Solving	Performance	Personalizing
			●			●

This means that as an instructional tool, Knowledge Cards can be used to meet either purpose or both depending on how you use it. For example, you can use Knowledge Cards relatively early in the learning process, thereby offering students a way to process new terms deeply through images and words. On the other hand, you might ask students to work in groups to create a set of Knowledge Cards at the end of a unit. In this context, Knowledge Cards are less about processing new content and more about developing a personally meaningful way to review and remember the important content in the unit. In both cases, however, elements of both processing and personalizing are present. The difference lies in which P gets the greater emphasis.

FIVE WAYS TO USE MATH TOOLS

So far, we have put the lion's share of our attention on the conceptual underpinnings for this book: What are tools? What are learning styles? What's the relationship between tools and styles? What's the relationship between tools and other critical factors in instructional decision making, from NCTM Process Standards to current instructional research to elements of quality lesson and unit design? We end this introduction by refocusing our attention on practical issues. So, before taking the plunge into the 64 tools that make up the remainder of this book, here are five different ways you can put these tools into effective practice.

1. **Try one out.** Every tool in this book is here because it makes a difference in the mathematics classroom. After all, a tool, by our standards, is only a tool if it addresses NCTM Process Standards, has a research base behind it, and plays a vital role in making lessons come to life. In other words, tools *work*. So, pick a tool, any tool, and watch what happens. Then try a few more. Before you know it, you and

your students will have your own personal favorites, and a new teaching and learning dynamic will be in full swing in your classroom.

2. *Use tools to help you meet a particular standard or objective.* Looking for ways to increase students' mathematical reasoning skills? Just use the Math Tools Matrices to find tools that address the NCTM Process Standard for Reasoning and Proof. Or maybe you're looking to build students' comprehension by helping them master critical concepts and terms or by infusing writing into the curriculum. Use the matrices to locate vocabulary tools or writing tools, respectively. The point is, the tools in this book can help you and your students meet those standards and objectives that matter most in your particular classroom, even as they shift throughout the year.

3. *Individualize instruction.* Remember that the tools in this book are organized according to the learning styles they naturally engage. So perhaps you're working with a highly creative Self-Expressive student who just can't seem to memorize and follow the steps in a critical problem-solving procedure. Try a tool like Math Recipes, which will allow students to make a creative comparison between cooking and problem solving and design a "recipe card" that outlines the steps in the procedure. If you want to help Interpersonal learners who wilt during independent seatwork to increase their proficiency as problem solvers, tap into their social nature with tools that either connect problem solving to the world beyond school (e.g., Real-World Connections or Who's Right?) or that challenge them to work as part of a productive problem-solving team (e.g., Cooperative Structures for Promoting Positive Interdependence). Style-based individualization works because every style has identifiable patterns of strength and weakness. Mastery learners may have no problem memorizing terms or following procedures, but often experience real difficulty with open-ended problems ("You mean there is no right answer?!") or high levels of abstraction. Understanding learners may be great at thinking their way through challenging problems, but their hearts often drop into their stomachs when they're asked to work as part of a team. As Robert Sternberg (2006) has shown, allowing students of mathematics to think and work in their strong styles gives them a much better chance at mastering key content and skills. Even better, when students' preferences are accommodated, they become more likely to try to stretch as learners, meaning you can use tools to teach to their strengths and to challenge them to try working in new styles that they might otherwise avoid.

4. *Differentiate instruction for the entire class.* While personalized instruction is a powerful teaching and learning model, the truth is that teachers do not often have the luxury of working like doctors, who see their patients one at a time. Teachers work with entire classes, groups of students brought together largely by virtue of their age and by scheduling logistics. The question that this model consistently raises is, *How can I work optimally with all of my students?*

Math tools, organized by style, make the work of differentiating instruction and assessment for every learner a manageable proposition. All you need to do is rotate the tools you use in your classroom across all four styles. That way, you can rest assured that your:

- Mastery learners are getting the routine and direction they thrive on while they develop their ability to think conceptually and creatively.
- Understanding learners have the opportunity to think logically and independently while growing their capacities as thoughtful team members.

- Self-Expressive learners get the chance to use their imaginations while learning how to manage and master mathematical procedures.
- Interpersonal learners can learn as part of a problem-solving community, where mathematics connects strongly to the real world, while they build their critical reasoning skills.

As you implement new tools, keep track of which styles you seem to favor and which tools seem to make the biggest impact among your students. And don't forget about Task Rotation (page 222), a strategy that uses tools from all four styles to create truly differentiated assessments.

5. *Design more powerful lessons, assessments, and units.* While tools work well on their own, they can also be used as "instructional building blocks." In Chapter 6, you'll find a set of strategies for selecting and combining tools to create larger designs, from lesson plans, to differentiated tests and assessments, to standards-based units of study. These strategies will help you develop a tools-based approach to thinking your way through the bigger picture of lesson planning and unit design.