Introduction

The Common Core State Standards are currently the driving force in educational change. Contained within the content standards are the Standards for Mathematical Practice. These Practices describe the varieties of expertise and abilities that mathematics educators should seek to develop in their students at all levels. These Mathematical Practices, if implemented, will significantly change student learning in mathematics by dramatically changing how teachers teach. Yet teachers are already feeling taxed to meet current demands.

Teachers face innumerable daily challenges in the course of fulfilling their jobs. Along with these daily challenges, teachers are also confronted by broader challenges that ebb and flow with societal and political trends. On this front, teachers are continually facing the “challenge de jour” approach to education. Rarely, it seems, are these types of challenges ever actually resolved. They merely become passé and are replaced by newer, more pressing challenges. Then, as is often the case, the original challenges resurface with new trendy terms. As a result of this tidal approach to reform, classroom instructional change has remained essentially inert.

In spite of this past history, some recent events have occurred that will unavoidably impact teachers and teacher leaders. With the adoption of the Common Core State Standards, a renewed focus has emerged that directly relates to student mathematics proficiency. The Common Core Standards, while impacting the nation, are state driven. States are devoting time, energy, and money into adopting and implementing the identified content, and more significantly, the Standards for Mathematical Practice. This bipartisan effort to improve mathematical learning is not going to fade away.

Leaders are being bombarded with terms and initiatives that end up pulling them in multiple directions, thus, creating lack of focus and clarity. Within this book, we show how all of the efforts directed at improving learning can be united, and when teachers and leaders work to implement the Practices, they are also accomplishing classroom formative
assessment and providing for diversity and rigor. This united approach ensures all students are successfully learning mathematics and prepares them to demonstrate learning of mathematics on the upcoming PARRC and Smarter Balanced assessments.

CCSS Content and Practices

To encourage and support this change in instruction, teachers and leaders must understand the Practices. Following is a list of the Common Core Standards for Mathematical Practice. We provide commentary on them throughout the book.

CCS Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Leaders and teachers need to know what they look like when incorporated into classroom instruction. This is especially true for student actions, since the primary focus for change in the last decades has been on teachers’ actions and teaching methods. With this shift from teacher focus to student focus, both teachers and leaders need assistance in reaching the goals of the CCSS. Moreover, teachers and leaders need to recognize the need for collaboration and to know how to effectively collaborate while working to implement the Practices. They need to share teaching strategies and lessons. What works for a particular practice? How much time is involved? How do we know what students are learning?

In the wake of Common Core, three important new trends have emerged. These trends, in conjunction with the Standards for Mathematical Practices, will challenge current teaching and learning processes and will fundamentally impact how teachers teach. These emerging trends all involve expanded use of technological advances:

1. Assessing students’ knowledge beyond multiple-choice formats.
2. Assessing students’ mathematical proficiency, thinking, and reasoning.
3. Tracking students’ knowledge gain over extended periods of time.
These trends demand that students are taught, and learn, rigorous mathematics, a phrase that we detail in Chapters 2 and 3. Teachers and leaders must step forward to take ownership of the potential impact of these trends to meet the demands of mathematical rigor.

The technological advances demand mathematical rigor for all students in every classroom. Students’ learning expectations in mathematics are expanding well beyond computational fluency because assessments are no longer bound to the constraints of multiple-choice test formats. The advances in assessment are happening now, as we have noted, and will continue. Teachers and leaders must be proactive by requiring students to think and reason mathematically to perform well on these challenging, open-ended assessments. Both teachers and leaders need to begin transforming classroom instruction with the goal of implementing rigor.

To date, however, mathematical rigor is rather loosely defined. While teachers have been pressed to become more rigorous, they have not been told what is actually expected as a result of being more rigorous. Rigor has been an elusive, unclarified expectation. Or worse, rigor has been narrowly defined and focused.

**A Clue to Rigor**

In our previous book, *The Common Core Mathematics Standards: Transforming Practice Through Team Leadership* (Hull, Harbin Miles, & Balka, 2011) we developed an easy-to-use Proficiency Matrix that is widely distributed and used. The Matrix lays out the Standards of Mathematical Practices called for in the CCSS and the expected outcomes for each level of student proficiency. We provide one row of the Matrix showing the three levels. In addition, strategies for implementing the Practices are provided in each cell (Grouping/Engaging, Encourage Reasoning).

<table>
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<tr>
<th>Students:</th>
<th>([I] = Initial) Reason with models or pictorial representations to solve problems. (Grouping/Engaging)</th>
<th>([IN] = Intermediate) Are able to translate situations into symbols for solving problems. (Grouping/Engaging)</th>
<th>([A] = Advanced) Convert situations into symbols to appropriately solve problems as well as convert symbols into meaningful situations. (Encourage Reasoning)</th>
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<tbody>
<tr>
<td>2</td>
<td>Reason abstractly and quantitatively</td>
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Teaching the CCSS content by incorporating the Practices as indicated in the proficiency levels of the Matrix makes it easier to achieve mathematical rigor.

The Matrix is a tool that was developed to assist teachers and leaders in effectively dealing with the necessary changes in teaching practices required by the Standards for Mathematical Practice and to steer their progress toward mathematically rigorous classrooms. It is designed to do the following:

- Focus collaborative conversations on implementation efforts.
- Serve as a guide for building effective lessons.
- Promote the Strategy Sequence of improving mathematical proficiency.

The Proficiency Matrix can play a critical role in helping teachers and leaders purposefully transforming classrooms to become more rigorous. The transformation to rigor is far more than just identifying content. The Practices focus learning related to students’ proficiency, thinking, reasoning, and depth of understanding. The Proficiency Matrix helps teachers identify learning progression in each mathematical practice. The Matrix, combined with formative assessment strategies and techniques, helps teachers assess where students are in their learning progression. With this knowledge, teachers can adjust their teaching to achieve advanced proficiency of the individual practice and help students succeed.

This book, *Realizing Rigor in the Mathematics Classroom*, focuses directly on effectively implementing the Standards for Mathematical Practice. Using the Matrix as a framework, we will show you how achieving mathematical rigor is the same journey as is incorporating the Practices. We provide a step-by-step guide for leaders and teachers to use, as well as supporting tools to assist in charting progress. We identify actions that indicate how a practice in the classroom begins at one level and then how that same practice deepens over time as both students and teachers gain proficiency.

If ever an opportunity existed to change mathematics instruction to address the learning needs of every student, this is that time.

**Outline of the Book**

The first section of the book lays the foundation of the impact and trends associated with the implementation of the Common Core. It also discusses and defines rigor. As a result of the Standards for Mathematical
Practice, student learning expectations in mathematics are expanding beyond computational fluency and moving more toward student thinking and reasoning. As such, technological advancements in assessing students’ knowledge, assessing students’ mathematical proficiency, thinking and reasoning and tracking students’ knowledge gain over time require a change in teaching practices. The Proficiency Matrix will be introduced at this point as a central tool to deal with implementation efforts.

The second section of the book focuses on potential issues or obstacles that will thwart teachers’ and leaders’ efforts to address the Common Core Standards for Mathematical Practices and increase student achievement. These include such things as differentiating instruction, monitoring classrooms, and using data. The entire book provides specific recommendations for meeting the challenges of the CCSS and effectively addressing the issues or obstacles.

The third section of the book provides a five-step solution process for implementing and sustaining the Standards for Practice and reaching the goal of providing a rigorous mathematics class.

The fourth section specifically addresses the necessary roles for teachers, coaches, and leaders to take to achieve implementation and rigor.

The last part of the book looks at how to sustain momentum for the implementation of the CCSS Mathematical Practices. It focuses on monitoring the many responsibilities of teachers, coaches, and school leaders to ensure successful implementation.

**How to Use This Book**

Because rigor is an outcome that is achieved by attending to specific inputs, attaining rigor takes time and focus. Mathematical rigor cannot be reached by maintaining our current instructional habits. Classroom instruction must incrementally change so that the climate and culture support rigor. Instructional change hinges on student thinking being made visible in every mathematics classroom by engaging students in instructional activities that require them to verbalize or demonstrate their understanding. The use of formative assessment information gathered from students through questioning, listening, or observing will help to immediately affirm or correct the displayed students’ understanding.

To implement the changes necessary to achieve the goals of the Practices and mathematical rigor, teachers and leaders should read and discuss this book to first understand formative assessment in the context of
mathematics. Once that is complete, they should begin to initiate use of the Proficiency Matrix and its recommended instructional strategies. Teachers and leaders should strive to undertake a cyclic pattern of implementation, monitoring, feedback, support, and improvement.

We provide numerous tools in the book for teachers and leaders to use to implement, monitor, refine, and sustain the Common Core Standards for Mathematical Practice. We have focused the book on productive conversations about student actions that directly relate to achieving the Practices, and the formative assessments necessary to impact change.

We emphasize students and the actions students should “engage” with mathematics. Any educator wishing to implement the Standards for Practice in a way that produces mathematical rigor will find our book beneficial, practical, and easy to use.