Introduction

Building the 21st-Century Mathematics Classroom

Imagine yourself as a second grader. In mathematics, you're adding, subtracting, multiplying, comparing fractions, and reading some basic graphs. Someone asks you, "Do you like math?" What would you say? Flash forward to fifth grade. The mathematics you're learning certainly has advanced, but so has your mind. How do you think you'd answer the question, "Do you like math?" Now, make one last jump. It's eighth grade, and you're making the transition from arithmetic to algebra. Once again, you're asked that simple question, "Do you like math?" What's your answer this time?

The fact is, studies show a disturbing trend in which "students in secondary school become increasingly less positive with regard to their attitude toward mathematics and their beliefs in the social importance of mathematics" (Wilkins & Ma, 2003, p. 58). For many students, this negative attitude becomes full-blown "math anxiety," an almost compulsive dislike of mathematics and mathematics instruction that emerges around fourth grade, reaches its peak in middle and high school (Scarpello, 2007), and sounds like this:

> I was terrified of math.
>
> I remember sitting in my seventh grade math class, staring at a quiz as if it were written in Chinese—it might as well have been a blank sheet of paper. Total brain freeze.
>
> Nothing made sense, I felt sick to my stomach, and I could feel the blood draining from my face. I had studied so hard, but it didn't seem to make any difference—I barely even recognized the math problems on the page.
>
> When the bell rang and my quiz was still blank, I wanted to disappear into my chair. I just didn't want to *exist.* (McKellar, 2008, p. xv)

These are the words of Danica McKellar, the actress who played Winnie Cooper on television's *The Wonder Years* and the author of *Math Doesn't Suck: How to Survive Middle School Math Without Losing Your Mind* *or Breaking a Nail.* While McKellar may be unique in that she became a famous actress before she was a teenager, her experiences as a middle school mathematics student are, sadly, all too common. For example, the classroom research that I have conducted with teachers and students over the last several years indicates that in third and fourth grade, almost 80% of students have positive attitudes towards mathematics and feel confident in their ability to succeed in mathematics. But as the mathematics curriculum becomes more difficult, more abstract, and more algebraic in middle school, the numbers change dramatically. By freshman year in high school, almost 50% of all students have developed an aversion to mathematics; they don't like it, they don't believe they're good at it, and many of them are proud to declare that they plan on taking the smallest number of mathematics courses in high school and beyond. This means that almost half of our students enter high school entertaining the dangerous idea that mathematics is a special realm for mathematicians and engineers, inscrutable to the average person and unnecessary for success in life.

This idea should give secondary teachers of mathematics the shivers. We know that mathematics is at the heart of so many things that affect everyone, from economics to technology, from the complexities of global marketing to the simple act of purchasing groceries. Mathematics, as Howard Gardner (1983, 1999, 2006) has shown us, is a vital form of human intelligence. Mathematics opens up career paths, empowers consumers, and makes all kinds of data meaningful—from basketball statistics to political polls to the latest trends in the stock market. Quite simply, we cannot afford to have so many secondary students who dread math class. We cannot allow half of our students to walk into a fast-moving, technological society looking to avoid confrontations with mathematics. For if we send an army of math-haters out into today's competitive global culture, we are short changing millions of students by severely limiting their chances of future success.

And yet, I have met many teachers of mathematics who are wondering openly if students really can be successful. "These kids hardly know basic mathematics. How can they be expected to do algebra?" is a common refrain from teachers in our middle schools. So what is the truth? Do we believe our students can be successful in mathematics or is the situation hopeless?

The good news is that research and experience both show that students' attitudes toward math and their problem-solving abilities are not fixed in place by the middle school years. In my 35 years of work in schools across the country, I have seen some truly remarkable changes in the way middle school students perceive mathematics and their ability to succeed in it. For example, I recently had the pleasure of working with a group of middle school mathematics teachers in Old Bridge, New Jersey. Together, we crafted a different kind of mathematics unit on three-dimensional figures. Instead of a test, we decided to build the unit around a summative assessment task (see Figure i.1 on page 3). This task required students to demonstrate just about everything they learned during the unit while also encouraging them to apply mathematics creatively. And in designing an instructional sequence to build the knowledge and skills students would need to succeed on the summative assessment task, we employed a variety of research-based strategies—strategies that we selected specifically

Final Task: A Monument to Learning

MathCorp has commissioned you to design and sketch a monument for a new math garden. The garden will have different sections, including sections devoted to important mathematicians and famous number sequences. The section of the garden they have asked you to design is the three-dimensional figure section.

Your task is to design and sketch a monument for the garden. The monument will be constructed of solid marble and must meet the following criteria:

1. In your design, you may only use the three-dimensional figures we learned about during our unit: *triangular prism*, *rectangular prism*, *triangular pyramid*, *rectangular pyramid*, *cylinder*, and *cone*.
2. You must include at least one of each of these three-dimensional figures in your design.
3. You must calculate the *volume* of your monument and show your work.
4. You must identify the total number of *bases*, *faces*, *edges*, and *vertices* within your monument.
5. You must include with your sketch a brief explanation of the thinking that went into your design. In your explanation, you must include at least ten critical vocabulary words from our investigation into three-dimensional figures and their volume.

FIGURE i.1 Summative Assessment Task

Source: Thoughtful Education Press. (2009). *Math Tools for Three-Dimensional Figures.* (Curriculum guide designed for the teachers of Old Bridge, New Jersey).

for their power to pique students' curiosity, actively engage students in learning, and speak to different styles of learners in the classroom.

When the teachers implemented their units in the classroom, the change in students' attitudes was palpable. Students were curious. They asked questions. They pursued difficult problems with vigor. Best of all, more students succeeded. In three of the four classrooms where the strategies were used, test scores rose by more than 3 full grade points. In one classroom, the average student score went from 73.71 to 81.41, an increase of over 10%—achieved with only one week of instruction. In the participating basic-skills classroom, test scores rose by 5.5 points, compared with an increase of only 1.5 points in the control group. By taking the time to engage students in the mathematics, the students were charged with learning; we also improved their comprehension, retention, and achievement levels.

This kind of change, of course, comes from teachers. And on this point, the research is sparklingly clear. A recent study tracking 3,000 seventh graders, for example, demonstrates that "teachers' choices of activities and mathematics problems can have a strong impact on the values that are portrayed in the classroom and on how students view mathematics and its usefulness" (Wilkins & Ma, 2003, p. 59). Of special note to middle school mathematics teachers is a meta-analytical study of 113 different studies suggesting that the middle school years are the most critical period for shaping students' attitudes towards mathematics and developing their confidence as mathematical problem solvers (Ma & Kishor, 1997).

So, how do middle school mathematics teachers use this critical time to engage and motivate more students to meet the new and higher demands of the 21st century, not to mention the challenges of expanding curriculums, state and national standards, school report cards, and greater expectations from colleges, government, and the public? The answer can be summed up in two simple but deep principles that drive this book and Ed Thomas's and John Brunsting's work in mathematics in general:

Effective mathematics instruction is *strategic.*

Effective mathematics instruction engages *all styles* of learners.

PRINCIPLE ONE: EFFECTIVE MATHEMATICS INSTRUCTION IS *STRATEGIC*

In what are two of the most comprehensive studies of the research behind various teaching strategies and their impact in the classroom, Robert Marzano (2007) and Robert Marzano, Debra Pickering, and Jane Pollock (2001) demonstrate conclusively that teaching strategies have a real and pervasive effect on student learning. Indeed, the evidence is clear: Classroom strategies like comparing and contrasting, developing and testing hypotheses, working cooperatively, creating visual representations, organizing information graphically, and using higher-order questions result in better performance and deeper learning among students. But as most teachers know, asking students to compare and contrast two time-distance-rate problems, for example, or to work cooperatively to solve a particularly rigorous problem may not result in the kinds of deep learning the research points to. It is in moments like these—when we apply research-based techniques only to experience a roomful of blank faces when what we were expecting was active engagement—that the gap between research and practice seems wider than ever. So, the question becomes, "How can I put this research into classroom practice so that it leads to a positive change in student learning?" To answer this question, let's look in on a classroom.

IN THE CLASSROOM, PART I

Situation: Alesandra Ciccio, a middle school mathematics teacher, has been teaching her students how to solve equations with one variable for the past week. Each day, Alesandra reviews the process, answers questions, provides in-class practice time, and assigns appropriate homework. She believes there is not much more she can do. Each day, when her students return to class, Alesandra finds they are still making many of the same mistakes. She is ready to test, move to the next unit, and admit that some of her students will never become proficient at the equation-solving process.

Applying a strategy: If Alesandra had a working knowledge of how and when to use teaching strategies for mathematics, she might have incorporated the Convergence Mastery strategy into her teaching. This strategy applied to Alesandra's situation would work as follows.

Once Alesandra realized that her students had reached an apparent plateau of proficiency, she would inform her students that they were going to participate in an engaging activity. She would prepare a series of five short quizzes on solving equations with one variable (see Figure i.2). Before each quiz, students would work cooperatively for 5 to 10 minutes to review and perfect the equation-solving process. Then, all students would be required to take the first quiz.

Quiz 1	Quiz 2	Quiz 3	Quiz 4	Quiz 5
1. $3x = 2x + 1$	1. $5x = 3x + 9$	1. $2z = z + 8$	1. $9a = 12a - 2$	1. $3c = 2c + 14$
2. $7b = 3b + 12$	2. $7c = 21 - 3c$	2. $6x = 4x - 2$	2. $5z = 24 - z$	2. $4a = 15 + a$
3. $4y = 8 - 2y$	3. $6y = 3 + 2y$	3. $4a = a + 6.5$	3. $3c = 2c + 9$	3. $9z = 11z - 18$
4. $5c = 2c + 9$	4. $11b = 5b + 6$	4. $y = 2y - 13$	4. $8b = 6b + 14$	4. $6x = 24 - 4x$
5. $6a = 4a + 10$	5. $3z = 4z - 5$	5. $5b = 3b + 7$	5. $2x = 4x - 9$	5. $y = 21 - 2y$

FIGURE i.2 Five Short Quizzes

At the end of the first quiz, students would cooperatively grade their solutions with Alesandra's help. Students who scored 100% would become permanent tutors and helpers and would exit the quiz-taking portion of the activity. Students who scored less than 100% would work cooperatively with the tutors and helpers to find their mistakes, correct them, and prepare for the next quiz. This process would continue until all five quizzes were taken. Since 100% success on a quiz is equivalent to an A in the grade book, students are highly motivated to communicate with each other, work cooperatively, and work hard to eliminate errors so they can take advantage of the immediate help and "retake" opportunities. As students progress through this process, they *converge toward mastery*.

What effect do you think Convergence Mastery would have in your classroom? Do you think students' mastery of the equation-solving process would improve as a result of the strategy?

Let's look in on another classroom where the students are having a different kind of problem.

IN THE CLASSROOM, PART II

Situation: Robert Gould is trying to curb his students' impulsivity as problem solvers. Too often, when Robert's students are faced with word problems, they will jump to solutions rather than engage in quality, presolution thinking and planning. This is especially worrisome to Robert since he knows that nearly one half of the items on his state's mathematics test are problems that students need to set up themselves.

Applying a strategy: Robert selects the strategy known as Math Notes because it is designed specifically to help students:

1. Identify the facts of the problem;
2. Determine exactly what the problem is asking;

(Continued)

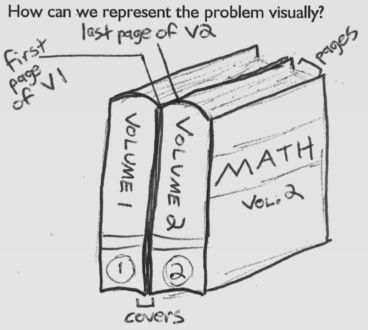
(Continued)

3. Represent the problem visually; and
4. Plan out the steps that need to be taken to solve the problem.

He begins by presenting this problem to students:

> *"Bookworm Problem"*
>
> *Volumes One and Two of a two-volume set of math books are next to one another on a shelf in their proper order (Volume One on the left, Volume Two on the right). Each front and back cover is $\frac{1}{4}$ inch thick and the pages portion of each book is 2 inches thick. If a bookworm starts at page 1 of Volume One and burrows all the way through to the last page of Volume Two, how far will the bookworm travel?*

Next, he asks students to take a minute and try to solve the problem as they normally do. As Robert suspects, nearly all the students answer impulsively, coming up with either *5 inches* ($2\frac{1}{2}$ inches for each book times 2) or $4\frac{1}{2}$ *inches* ($2\frac{1}{2}$ inches for each book minus $\frac{1}{2}$ inch for the front and back cover). That's when Robert introduces and models Math Notes. Using the same problem, Robert shows students how he thinks through and sets up the problem on a Math Notes organizer (Figure i.3).

The Facts	**The Steps**
What are the facts? • Volumes one (V1) and two (V2) are next to each other • V1 on the left, V2 on the right • Covers are ¼-inch thick • Total pages are 2-inches thick • Bookworm starts at page 1 of V1 and burrows to last page of V2 What is missing? • Distance bookworm traveled	What steps can we take to solve the problem? 1. The bookworm burrowed through two covers. 2. Each cover is ¼-inch thick. 3. Bookworm burrowed through two covers but no pages. 4. Add thickness of covers plus thickness of pages.
The Questions	**The Diagram**
What questions need to be answered? • How far did the bookworm travel? Are there any hidden questions that need to be answered? • How many pages did the bookworm burrow through? • How many covers did the bookworm burrow through?	How can we represent the problem visually? 

The Solution

covers + pages = total distance traveled

$\frac{1}{4} + \frac{1}{4} + 0 = \frac{1}{2}$ inch

FIGURE i.3 Completed Math Notes Organizer

What students see very clearly as a result of Robert's use of Math Notes is that without a strategy for breaking down, attacking, and visualizing difficult word problems, they are likely to miss essential information or misinterpret what the problem is asking them to do.

Over the course of the year, students keep a notebook of problems they've solved using Math Notes. This way, they can refer back to their notebooks and look for models they can use whenever they come across new problems.

Convergence Mastery and Math Notes are only two of the 21 research-based teaching strategies that Ed Thomas and John Brunsting lay out in this book. Convergence Mastery is, as its name suggests, a Mastery strategy—a strategy focused on helping students remember mathematical procedures and practice their computational skills. But mathematics, of course, is about more than memory and practice. It is also about asking questions, making and testing hypotheses, thinking flexibly, visualizing concepts, working collaboratively, and exploring real-world applications. To accommodate this cognitive diversity, the strategies in this book are broken up into five distinct categories. Four of these categories—Mastery, Understanding, Self-Expressive, and Interpersonal—develop specific mathematical skills. The fifth category, Multistyle strategies, contains strategies like Math Notes, strategies that foster several kinds of mathematical thinking simultaneously. The following map (Figure i.4) explains these five categories.

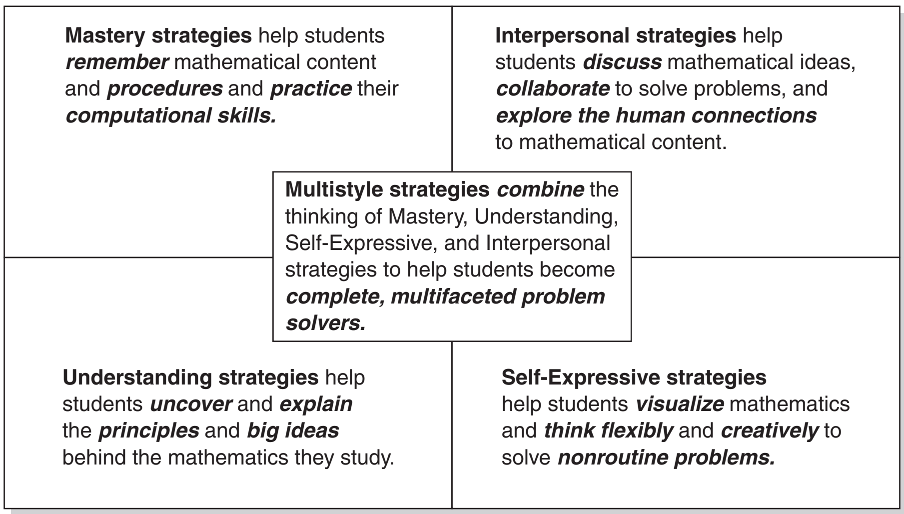


FIGURE i.4 Map of Mathematical Strategies

Each of the strategies in these five categories represents a different kind of thinking, a different way of interacting with mathematical content, a different opportunity to grow as a learner and problem solver. Take just one of these ways of thinking away, and you really don't know mathematics. Think about it: If you can't compute accurately (Mastery), explain mathematical concepts (Understanding), find ways to solve nonroutine problems (Self-Expressive), or explore and discuss real-world applications with fellow problem solvers (Interpersonal), then you don't have the complete picture; and without a complete picture, you don't *really* know mathematics. This simple but often-overlooked idea—that mathematical learning and problem solving require the cultivation of different kinds of thinking—brings us to the second way that this book will help you and your students achieve higher levels of success: *learning styles.*

PRINCIPLE TWO: EFFECTIVE MATHEMATICS INSTRUCTION ENGAGES *ALL STYLES* OF LEARNERS

Let's listen in on two secondary students who were asked the same question:

"Who was your favorite mathematics teacher and why?"

> ***Alisha:*** My favorite math teacher so far has definitely been Ms. Tempiano. She really taught, and by that I mean she was very clear about explaining what we were learning and always showed us exactly how to do it. Whenever we learned a new skill or a new technique, not only would she review the steps, she would work with us to develop a way to help us remember how to apply the steps, like the acronym "Please Excuse My Dear Aunt Sally" for remembering the order of operations. Once we knew the steps, she would let us practice the steps to different problems. Sometimes we practiced alone and sometimes we practiced in groups, but Ms. Tempiano always walked around the room and worked with us like a coach. I loved getting feedback right away. That really helped me when she would walk around and watch what we were doing and help us with any problems we were having.
>
> ***Ethan***: I didn't really think I liked or was good at math before I had Mr. Hollis for Algebra I. He did this thing called "Problem Solving Fridays." Every Friday, we focused on what he called "nonroutine" problems, which were basically these really cool problems about things like building bridges or developing a new lottery game, problems that didn't have simple answers. So, we had to experiment, try different things out—you know, get creative—to see how we might be able to find a solution. Actually, I knew I would like Mr. Hollis on the first day of class.
>
> I was a freshman and math was first period. I walked in expecting the same old thing: worksheets, the odd problems, quizzes. But instead, Mr. Hollis spent the first day on metaphors! He challenged us to create a metaphor for the problem-solving process. I showed how each step in the problem-solving process was like one of the stages in human digestion. It was really cool—I showed how you "chew" and "breakdown" and "process" both equations and food. The class loved it. And you know what else? I never forgot the steps in solving equations after that.

Almost immediately, we can see that Alisha and Ethan treat mathematics very differently. Alisha is attracted to problems that have clear solutions.

Ethan, on the other hand, gets excited about nonroutine problems where finding a solution requires experimentation and flexibility.

Alisha solves problems by selecting an algorithm and applying it step by step, while Ethan's problem-solving process is one of generating and exploring alternatives. As far as teachers of mathematics go, Alisha prefers one who is clear about expectations, models new skills, allows students to practice the skills, and provides regular feedback and coaching along the way. From Ethan's point of view, an ideal mathematics teacher allows students to explore the content through the imagination and creative problem solving. Finally, and most significantly, each student sees different purposes for learning and using mathematics. For Alisha, mathematics represents structure and stability, a set of failsafe procedures that can be used again and again to find correct solutions. Ethan, of course, would disagree. For him, mathematics is a medium for expressing powerful ideas and creating new and interesting products—a kind of intellectual playground full of possibilities, unseen connections, and fascinating applications. The differences in how these two students experience and approach mathematics are the result of *learning styles.*

Learning styles come from psychologist Carl Jung's (1923) seminal work on the human mind. Jung, one of the founding fathers of modern psychology, discovered that the way we take in information and then judge the importance of that information develops into different personality types. Working from Jung's foundational work on personality types, Kathleen Briggs and Isabel Myers (1962/1998) later expanded Jung's model to create a comprehensive model of human difference, which they made famous with their Myers-Briggs Type Indicator (1962/1998).

Since the development of the Myers-Briggs Type Indicator, new generations of researchers have worked to apply and adopt the personality-types model to the specific demands of teaching and learning. Bernice McCarthy (1982), Carolyn Mamchur (1996), Edward Pajak (2003), Gayle Gregory (2005), and Harvey Silver, Richard Strong, and Matthew Perini (2007) are some of the key researchers who have helped educators convert and expand the insights of Jung and Myers and Briggs into a more practical and classroom-friendly model of cognitive diversity—learning styles.

A few years back, I initiated a new research study with one of the authors of this book, Ed Thomas. Our goal was to make a deep connection between mathematics and learning styles. We reviewed the research on learning styles, worked with teachers of mathematics and their students in classrooms, and developed a new instrument for assessing students' mathematical learning styles—*The Math Learning Style Inventory for Secondary Students* (Silver, Thomas, & Perini, 2003). Out of our work, we identified four distinct styles of mathematical learners, which are outlined in Figure i.5 on page 10.

It is important to remember that no student—no person—is a perfect representative of a single style. Learning styles are not pigeonholes; it is neither possible nor productive to reduce this student to a Self-Expressive

Mastery Math Students	Interpersonal Math Students
Want to learn practical information and set procedures. ***Like math problems that*** are like problems they have solved before and that use algorithms to produce a single solution. ***Approach problem solving*** in a step-by-step manner. ***Experience difficulty when*** mathematics becomes too abstract or when faced with nonroutine problems. ***Want a math teacher who*** models new skills, allows time for practice, and builds in feedback and coaching sessions.	***Want to*** learn math through dialogue, collaboration, and cooperative learning. ***Like math problems that*** focus on real-world applications and on how mathematics helps people. ***Approach problem solving*** as an open discussion among a community of problem solvers. ***Experience difficulty when*** instruction focuses on independent seatwork or when what they are learning seems to lack real-world application. ***Want a math teacher who*** pays attention to their successes and struggles in mathematics.
Understanding Math Students	**Self-Expressive Math Students**
Want to understand why the math they learn works. ***Like math problems that*** ask them to explain, prove, or take a position. ***Approach problem solving*** by looking for patterns and identifying hidden questions. ***Experience difficulty when*** there is a focus on the social environment of the classroom (e.g., on collaboration and cooperative problem solving). ***Want a math teacher who*** challenges them to think and who lets them explain their thinking.	***Want to*** use their imagination to explore mathematical ideas. ***Like math problems that*** are nonroutine, project-like in nature, and that allow them to think outside the box. ***Approach problem solving*** by visualizing the problem, generating possible solutions, and exploring among the alternatives. ***Experience difficulty when*** mathematics instruction is focused on drill and practice and rote problem solving. ***Want a math teacher who*** invites imagination and creative problem solving into the mathematics classroom.

FIGURE i.5 Four Styles of Mathematics Students

Source: Silver, H. F., Thomas, E., & Perini, M. J. (2003). *Math Learning Style Inventory for Secondary Students*. (p. 8).

learner or that student to an understanding learner. Various contexts and types of problems call for different kinds of thinking, and all students rely on all four styles to help them learn mathematics. However, it is equally true that people tend to have style preferences; like all people, each student will usually show strength in one or two styles and weakness in one or two others. What all this means is that learning styles are the key to motivating students, improving their attitudes toward mathematics, and helping them experience higher levels of success. Tapping into the power of learning styles is a matter of building on students' strengths by accommodating their preferred styles while simultaneously encouraging them to stretch their talents and grow as learners by developing less-preferred styles.

Recent research conducted by Robert Sternberg (2006) shows that rotating teaching strategies to reach all styles of learners is about more than being fair; it's about being effective. Sternberg and his colleagues conducted a remarkable series of studies involving diverse student populations from around the world. As part of these studies, students were taught mathematical content in five different ways. Some students were taught using

1. A memory-based approach emphasizing factual recall;
2. An analytical approach emphasizing critical thinking;
3. A creative approach emphasizing imagination;
4. A practical approach emphasizing real-world applications; and
5. A diverse approach incorporating all four approaches.

Which group of students who participated in these studies do you think did best? Hands down it was the students who were taught using all four approaches. They did better on objective tests, and they did better on performance assessments. From these studies, Sternberg (2006) concludes, "even if our goal is just to maximize students' retention of information, teaching for diverse styles of learning still produces superior results. This approach apparently enables students to capitalize on their strengths and to correct or to compensate for their weaknesses, encoding material in a variety of interesting ways" (pp. 33–34).

So, how do we accomplish this goal of teaching for diverse styles? Take another look at our map of strategies below (Figure i.6).

What the map shows us is how styles and strategies come together, the place where they meet. Accommodating students' strong styles and

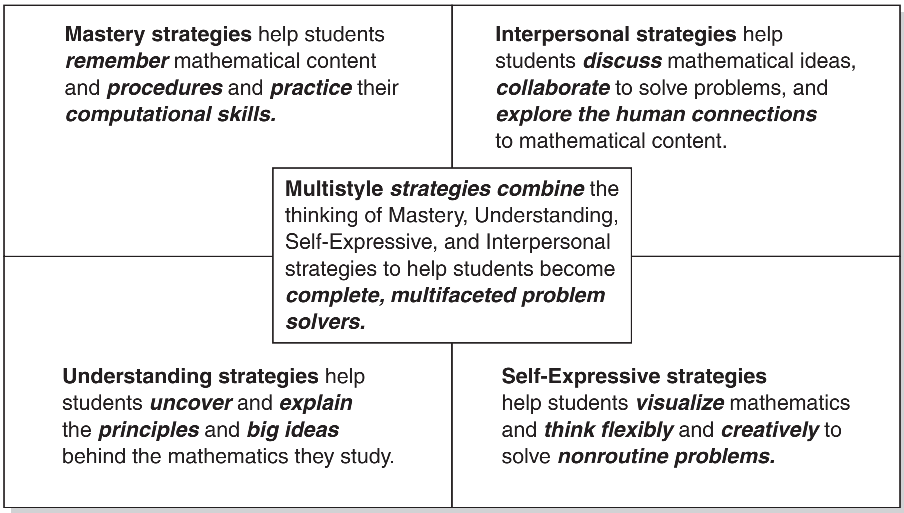


FIGURE i.6 Map of Mathematical Strategies fostering their weaker styles requires us to vary the strategies we select and use in our classrooms. When you use a Self-Expressive strategy, for example, not only are you inviting your creative students who think mathematics is too black and white into the learning process, you are also challenging all of your "procedure whizzes" to step back and think about mathematics in a new and illuminating way. The same is true for the Mastery, Understanding, and Interpersonal strategies: The different kinds of thinking required by each style of strategy will engage some students and challenge others, while the Multistyle strategies combine the thinking of all four styles inside a single strategy.

The key to making all of this work in the classroom is *rotation.* Use all five types of strategies regularly. Keep track of what styles you use and when. Here's an experiment: If a concept seems to be eluding students, try using a strategy like Metaphorical Expression (Self-Expressive) or Compare and Contrast (Understanding). If students of all styles need to work on complex problem solving, try a Multistyle strategy like Math Notes.

Remember that good problem solving requires all four styles of thinking; therefore, teaching students how to become good problem solvers will require you to rotate around the "wheel of style."

THE 21ST-CENTURY CLASSROOM: WITHIN OUR REACH

What Ed Thomas and John Brunsting have found, through decades of teaching mathematics and conducting professional development seminars for teachers of mathematics, is that building a 21st-century mathematics classroom means "making students as important as standards." But rhetoric is one thing; a mathematics classroom that is humming with the thought of actively engaged students is quite another.

What Ed and John show is that getting the classroom we all wish for is not pie-in-the-sky idealism. Thankfully, building a 21st-century mathematics classroom does not require us to reinvent ourselves or our beliefs. By developing a working knowledge of the research-based strategies in this book and by rotating them so that you accommodate and grow the learning styles of all your students, you can increase significantly the power of your teaching and your students' learning.

We know this book will be an important tool in developing such a classroom.

Tr. Harvey F. Silver, EdD