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TEACHING AND ASSESSING

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FIGURING OUT FLUENCY: REPLACING FALLACIES WITH TRUTHS

GOOD (AND NECESSARY) BEGINNINGS FOR FLUENCY

CONCEPTUAL UNDERSTANDING

PROPERTIES AND UTILITIES FOR STRATEGIC COMPETENCE

DISTANCE FROM A 10

DECOMPOSING NUMBERS FLEXIBLY

PART–PART–WHOLE

SKIP COUNTING

DISTRIBUTIVE PROPERTY

GOOD BEGINNINGS: BASIC FACT FLUENCY

#1 A STRATEGY-BASED APPROACH LEADS TO "BETTER"-LEARNED FACTS

#2 FACTS STRATEGIES ARE NEEDED BEYOND BASIC FACTS

#3 LEARNING BASIC FACT STRATEGIES DEVELOPS MATHEMATICAL AGENCY

COMPUTATIONAL ESTIMATION

COMPUTATIONAL ESTIMATION STRATEGIES
SEVEN SIGNIFICANT STRATEGIES FOR DEVELOPING FLUENCY

DEVELOPING STRATEGIC COMPETENCE

1. COUNT ON/COUNT BACK (ADDITION AND SUBTRACTION)
   WHAT IT IS
   VISUALS TO SUPPORT
   SITUATIONS THAT CONNECT

2. MAKE TENS (ADDITION)
   WHAT IT IS
   VISUALS TO SUPPORT
   SITUATIONS THAT CONNECT

3. USE PARTIALS (ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION)
   WHAT IT IS
   VISUALS TO SUPPORT
   SITUATIONS THAT CONNECT

4. BREAK APART TO MULTIPLY (MULTIPLICATION)
   WHAT IT IS
   VISUALS TO SUPPORT
   SITUATIONS THAT CONNECT

5. HALVE AND DOUBLE (MULTIPLICATION)
   WHAT IT IS
   VISUALS TO SUPPORT
   SITUATIONS THAT CONNECT

6. COMPENSATION (ADDITION, SUBTRACTION, AND MULTIPLICATION)
   WHAT IT IS
   VISUALS TO SUPPORT
7. USE AN INVERSE RELATIONSHIP (SUBTRACTION AND DIVISION)

WHAT IT IS

VISUALS TO SUPPORT AND SITUATIONS THAT CONNECT

EXPLICIT STRATEGY INSTRUCTION

FIGURING OUT FLUENCY: SEVEN SIGNIFICANT REASONING STRATEGIES

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AUTOMATICITY BEYOND THE BASIC FACTS

AUTOMATICITY

LEARNING PROGRESSION FOR AUTOMATICITY WITH BASIC FACTS

AUTOMATICITY AND FLUENCY

TIPS FOR DEVELOPING PROCEDURAL AUTOMATICITIES

PROCEDURAL AUTOMATICITIES

BREAKING APART ALL NUMBERS THROUGH 10

BASE 10 COMBINATIONS

USING 25s

USING 15s AND 30s

DOUBLING

HALVING

FRACTION EQUIVALENTS WITHIN FRACTION FAMILIES

CONVERSIONS BETWEEN COMMON DECIMALS AND FRACTIONS

FIGURING OUT FLUENCY: PROCEDURAL AUTOMATICITIES

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FLUENCY PRACTICE IS NOT A WORKSHEET

CHARACTERISTICS OF QUALITY FLUENCY PRACTICE

QUALITY PRACTICE IS FOCUSED

QUALITY PRACTICE IS VARIED

QUALITY PRACTICE IS PROCESSED

QUALITY PRACTICE IS CONNECTED

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