What Your Colleagues Are Saying . . .

“Kathryn Chval and her colleagues exquisitely give the readers opportunities to see inside a classroom with multilingual students, develop empathy, and deeply understand effective practice. The book’s engaging format provides questions for reflection and strategies to try out, connects research to practice, and compels readers to position students for success.”

—Nora G. Ramirez
Executive Director,
TODOS: Mathematics for ALL

“This is the book I’ve been waiting for! It is powerful. It brings together often-separate critical ideas for teaching multilingual students and weaves them with in-depth explorations of classrooms. We meet remarkable teachers, whose success we can learn from, which will help us reimagine what’s possible.”

—Lena Licón Khisty
Emerita, University of Illinois Chicago

“A must-read! This book is an excellent resource to closely examine mathematics instruction that affirms multilingual learners’ identities, competencies, and growth as learners of mathematics. Far too often multilingual children and their families are positioned in deficit ways that lead to limited learning. This book does the opposite. It seamlessly blends practice and research for a comprehensive look at exemplary mathematics teaching that leverages children’s multiple linguistic, cultural, and mathematical strengths. The book offers practical tools and guidance to enhance mathematics instruction, nurture student relationships, and create strong partnerships with families to support and advance multilingual learners in mathematics.”

—Julia Aguirre
Faculty Director of Teacher Credential Programs,
School of Education,
University of Washington Tacoma

“This book goes beyond the typical support of the academic language of mathematics for English learners. It provides an in-depth perspective on being more culturally inclusive of English learners and allows educators to reflect on their instructional methodologies in mathematics.”

—Alexander L. Tai
Teacher and English Learner Specialist,
Columbia Public Schools
“This book celebrates the brilliance of multilingual learners while also providing evidence-based strategies for teachers. The included cases and activities provide a solid foundation for teachers’ growth and exploration into teaching mathematics with multilingual students. This book will help teachers and teacher educators engage in meaningful and humane mathematics instruction with students.”

—Zandra de Araujo
Associate Professor, College of Education, University of Missouri

“This inspiring volume provides resources for mathematics teachers to support mathematics learning for English learners. Using four central principles—assets, empathy, practice, and research—to base the strategies and an impressive array of materials, including student work, the volume illustrates multiple approaches to providing English learners with opportunities to learn important mathematics with understanding.”

—Judit Moschkovich
Professor, University of California, Santa Cruz

“Teaching Math to Multilingual Students, Grades K–8: Positioning English Learners for Success takes an asset-based approach toward developing multilingual learners in the classroom. This book clearly demonstrates the nuances of analyzing the mathematical work of multilingual learners while providing examples and strategies for giving useful feedback that is applicable to all learners. Fostering a culture of writing in the mathematics classroom is explicitly taught through a variety of strategies, activities, and teacher practices. Topics such as culturally relevant contexts, crafting language, and family involvement serve to round out this text and provide teachers with a solid resource to support multilingual learners in a layered, thoughtful way.”

—Renee Rowan
Second-Grade Teacher, Skokie, Illinois

“Wondering how to support multilingual learners beyond broad, generic suggestions? This book is it! Through true vignettes, transcripts, pictures, and videos, these authors literally show how to support multilingual learners, while engaging you in developing your own capacities to do so. The chapter on positioning learners as leaders is a must-read for every educator! I can’t wait to use this book in my work with students and teachers.”

—Jennifer Bay-Williams
Author and Professor, University of Louisville
“This book is an excellent resource for opening doors of access to mathematics for multilingual students, particularly those multilingual students who are, in the authors’ words, ‘silent spectators’ of classroom lessons. *Teaching Math to Multilingual Students, Grades K–8* offers strategies and resources that are both research-based and tried, personalized, and polished in real classrooms. The images from those classrooms are compelling, underscoring the importance of an asset-based mentality in teaching multilingual students.”

—Mark Driscoll  
Coauthor of *Mathematical Thinking and Communication: Access for English Learners*

“This book is a must-have for anyone working with multilingual learners in mathematics. The authors push the reader to reflect through questions and prompts and to take action by trying out the strategies suggested. The authors’ deep respect for and asset-based view of multilingual students and their mathematical ideas are evident throughout the whole book. Of particular note is the attention paid to the role of families in the mathematics education of multilingual learners.”

—Marta Civil  
Professor of Mathematics Education  
and Roy F. Graesser Chair—Department of Mathematics,  
The University of Arizona

“This groundbreaking book offers practical, research-informed strategies and activities that support all learners. Chval and her colleagues’ innovative approach positions multilingual learners as potential classroom leaders in challenging mathematics learning. Even newcomers to English are invited to draw on all their meaning-making resources to participate in the mathematics classroom.”

—Mary J. Schleppegrell  
Professor of Education,  
University of Michigan, Ann Arbor

“This book provides a powerful tool for teachers as they engage multilanguage learners in language and mathematics. Each chapter is a wonderful compilation of research and practice that unpacks the strategies that will empower teachers to build upon the unique strengths and knowledge that multilingual students bring to the classroom.”

—Amy Stephens  
Senior Program Officer,  
Board on Science Education,  
The National Academies of Sciences, Engineering, and Medicine
Teaching Math to Multilingual Students
The Book at a Glance

Multilingual learners deserve the same social and academic opportunities to learn and be successful as their English-speaking peers. This book takes an asset-based, empathetic, practical, and research-based approach to help you position multilingual learners as leaders in your mathematics classes so that they may strive for success. You will be aided in your journey through:

STOP AND THINK
Stop and think about Xiao’s experience in this situation.

• How would you feel if you were Xiao?
• How can positive positioning actually work against students?

REFLECTING ON NABIL ABADI’S EXPERIENCE
When we examine Nabil Abadi’s experience, we can see that his teacher spends time observing Nabil’s interactions first. She does not make assumptions about what might be causing Nabil’s frustration, and she does not allow a negative storyline such as “troublemaker” to be instituted. Instead, she observes and notices how particular students dominate small-group talk and activities. Recognizing that this is harmful for every student, she uses the power of her position as the teacher to make space for Nabil to speak. By doing so, she positions Nabil as someone with important contributions to make and one who has ideas that others should listen to and consider. This also provides opportunities for her to learn about Nabil’s mathematical sensemaking and give Nabil opportunities to use English to share his mathematical ideas.

Reflection Questions to help you examine your teaching practice.

• Imagine you were to record and study your teaching. What would you find about your positioning of multilingual learners?
• What strategies will you use more often to position multilingual learners as leaders in your classroom?
• What storylines are present in your mathematics classroom? Are there storylines you want to alter? If so, how will you alter them or promote new ones?

Instructional Strategies to assess student understanding, partner students, appropriately challenge them, identify and draw out their strengths, and encourage and motivate student participation.

STRATEGIES FOR PROMOTING CLASSROOM DISCOURSE
Mercer (1995, p. 32) identified the following strategies that promote and initiate classroom discourse:

› Make a declarative (open-ended or provocative) statement that invites a rejoinder or disagreement;
› Invite elaboration (“Could you say a bit more about that?”);
› Admit perplexity when it occurs, whether about the topic itself or about a pupil’s contribution to it;
› Encourage questions from pupils (rare in many classrooms); and
› Maintain silence at strategic points (Dillon [(1982), another classroom researcher,] suggests that three to five seconds may be enough to draw in another pupil’s contribution or encourage the previous speaker to elaborate on what was said).
Try It! 4.2
Choose Partners for Multilingual Learners

Pat and Sam are two new students who will join your class tomorrow. Here is some information about each of them.

Pat is a third-grade student who has just joined your class after recently moving to Missouri from New Mexico, where Pat’s family lived for two years. Pat’s family relocated to be closer to extended family and possible new financial opportunities as the family’s financial situation was bleak in New Mexico. Pat is the middle child of five, who all live with their mother. Since moving to Missouri, Pat has not adjusted well socially and has been unable to make friends. Pat is very quiet in class, appears timid, and does not participate in class discussions.

Sam is also a third-grade student who has just joined your class after moving to Missouri from Mexico. Sam’s family relocated for new job opportunities. In Mexico, Sam’s family was financially stable and had the privilege to travel annually. Sam’s family is composed of a younger sister, Nancy, and both parents. Since moving to Missouri, Sam has had trouble integrating into the school community and making friends. In class, Sam has not performed well academically and has picked a fight with another student.

• What qualities would you look for when selecting partners for Pat and Sam?
• Why are these qualities important for each of these specific students?

Vignettes and Transcripts of real conversations between teachers and students as well as between teachers and the authors themselves.

Try It! Activities that invite you to apply the strategies.

Try It! 4.2
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• What qualities would you look for when selecting partners for Pat and Sam?
• Why are these qualities important for each of these specific students?

Vignettes and Transcripts of real conversations between teachers and students as well as between teachers and the authors themselves.
Teaching Math to Multilingual Students

Grades K–8
This book is dedicated to Sara Martínez, an exceptional teacher who was the impetus and inspiration for the work.

We also dedicate this book to multilingual learners and their families across the United States and the world because they enrich our schools, communities, and cultures with their diverse perspectives, experiences, and knowledge. We are grateful to the families who graciously opened their homes so others could learn from them.
Teaching Math to Multilingual Students

Positioning English Learners for Success

Grades K–8

Kathryn B. Chval, Erin Smith, Lina Trigos-Carrillo, and Rachel J. Pinnow

A Joint Publication
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Note From the Publisher: The authors have provided video and web content throughout the book that is available to you through QR (quick response) codes. To read a QR code, you must have a smartphone or tablet with a camera. We recommend that you download a QR code reader app that is made specifically for your phone or tablet brand.
Preface

Throughout our careers, we have had the privilege of collaborating with and researching inspirational teachers. Ms. Sara Martínez is one of those exceptional teachers. She opened her elementary mathematics classroom to us so that we could learn from her practice. We are grateful that the results from that first study in her classroom became the seed for robust and innovative research that was replicated in other classrooms in different grade levels and contexts.

Ms. Martínez is a teacher who establishes conditions for student success, where every child is respected and challenged, has the flexibility to solve mathematics problems in several ways, and is given the opportunity to communicate mathematical thinking in her classroom. Everyone who enters Ms. Martínez’s classroom is mesmerized by what her fifth graders can do mathematically. Ms. Martínez creates a classroom community whose culture is characterized by respectful challenge, agreement and disagreement, and argument. The students listen closely to each other’s ideas, build on each other’s work, and can complete a peer’s problem-solving strategy at a moment’s notice. In this book, we share examples of Ms. Martínez’s practice during her 20th year of teaching in Chicago. We recorded lessons with her class of 24 fifth graders during a year when her school reported 96.8% of the students as low-income; 96.9% as Hispanic; 46% with limited English proficiency (the term the state uses; however, not a term we endorse); and a mobility rate of 21.5%.

During our professional development sessions with teachers around the country, we have shared lessons learned from inspirational teachers such as Ms. Martínez. As a result, we consistently receive requests for specific strategies and materials that facilitate the engagement of children learning mathematics in languages that differ from their native languages—in other words, multilingual learners. This book was born out of that demand.

Multilingual learners deserve the same social and academic opportunities to learn and be successful as their English-speaking peers. All students should learn how to interpret the meaning of problems, make conjectures, analyze mathematical thinking and solutions, monitor and evaluate their progress, and understand the approaches of others in comparison with their own. These expectations emphasize the vital role of language and communication in solving mathematical problems, including the different domains of language (i.e., reading, writing, speaking, and listening) in developing mathematical thinking, and demonstrating knowledge in classroom interactions. As teachers, we must facilitate access, participation, and success for multilingual learners. To do this effectively, we must recognize that multilingual learners require opportunities to learn content while simultaneously developing a new language.
OUR UNIQUE PERSPECTIVE

This book has four underlying principles. This book is . . .

› **Asset-based:** Multilingual learners “bring new perspectives and resources to the classroom through their participation and sharing of experience that can benefit their peers” (National Academies of Sciences, Engineering, and Medicine, 2018a, p. 21). Multilingual learners are intellectual leaders of classrooms. Everyone can learn from their complex knowledge and experience when we position students’ language and culture as valuable resources for learning (Ladson-Billings, 2014; Orellana, 2016). Therefore, we encourage you to draw on students’ academic success, social and cultural identities, and family participation (i.e., assets or strengths) to make your lessons more interesting for all of the students in your classroom (Kobett & Karp, 2020; Paris, 2012).

› **Empathy-based:** We focus on developing empathy by asking you to imagine yourself in the shoes of multilingual learners, parents/families of multilingual learners, or teachers of multilingual learners. We set up situations and provide questions that facilitate reflection and the consideration of different perspectives. For example, you will consider times when you were reluctant to speak in public or share your thinking, and when you were in situations where information was confusing due to the cultural context such as viewing a cricket match.

› **Practice-based:** Each chapter includes content from our studies of teaching practice in elementary classrooms. During our research, we recorded and transcribed mathematics lessons, interviews and lesson planning sessions with teachers, and interviews with multilingual learners and their parents. We also collected copies of student work. We integrate different examples of teaching practices and strategies designed to illustrate diverse, effective ways of teaching. You will find examples of teachers with different levels of teaching experience, ranging from 1 year to 20 years, from a variety of school settings, including rural and urban contexts.

› **Research-based:** Too often, we hear deficit perspectives and narratives about teaching and teachers—in other words, what teachers do not know or cannot do. As we researched classroom teaching, we identified what the participating teachers wanted to learn about enhancing their teaching practice and then facilitated that journey along with them. We drew from research on how people learn (National Academies of Sciences, Engineering, and Medicine, 2018b) to build on what teachers already knew to develop the specialized knowledge and competencies to teach mathematics to multilingual learners. Along the way, we know we learned more from the teachers, parents, and multilingual learners than we could have imagined. The insights gleaned from this research make this book unique.
AUDIENCE

A range of audiences will benefit from the content of this book, including novice and experienced K–8 teachers; mathematics coordinators, coaches, and supervisors; curriculum coordinators; mathematics teacher educators; professional development facilitators; and faculty in teacher preparation programs. We encourage you to facilitate conversations about the ideas in this book across your schools, organizations, and communities. A key factor in teaching multilingual learners is to enhance the cultural environment of schools and to foster the critical conversations necessary to build strong schools. In the development of this book, we discussed the chapters with preservice teachers, English as a second language (ESL) and mathematics coordinators, practicing teachers, and administrators over the course of several years. Their contributions and insights have helped create a book that addresses the needs of a wide range of stakeholders.

ORGANIZATION

Working with multilingual learners is complex. In this book, you will not find quick fixes. Instead, each chapter presents a different aspect of teaching mathematics to multilingual learners that must be thoroughly considered. The strategies in the book will help you draw out the strengths and knowledge of multilingual learners and other students. These practices will profoundly affect every student in the classroom so that when they are adults, they too will function with an asset-based mentality when meeting people who differ from them or who are marginalized by society.

Within each chapter, you will encounter

- **reflection questions** to help you examine your teaching practice;
- **strategies** to assess student understanding, appropriately challenge students, partner students, identify and draw out students’ strengths, and encourage and motivate student participation;
- **Try It! activities** that invite you to apply the strategies; and
- excerpts from **transcripts** of conversations between teachers and students as well as between teachers and ourselves.

After reading this book, you will be able to

- support the development of mathematics and language for multilingual learners;
- enhance curriculum materials to ensure they are challenging and accessible for multilingual learners; and
- position multilingual learners for success as individuals, in groups, and in whole-class settings.
We encourage you to think about the following questions as you read each chapter:

- How am I ensuring the academic success of multilingual learners?
- In what ways am I valuing, sustaining, and learning from multilingual learners and their families’ heritage, knowledge, and culture?
- How am I working with multilingual families so that they are partners in educating their children?
Acknowledgments

This book was a labor of love. It would not have been possible without the commitment, generosity, and dedication of our collaborators, funders, partners, and family. We thank each of you for your time, talent, trust, and intellect.

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We are grateful for our school collaborators, including district and school administrators (i.e., superintendents, principals, ESL coordinators, curriculum coordinators, assessment directors, translators, and office professionals who greeted us during each visit). A special thank-you to the teachers who spent countless hours with us. They trusted us with their most valued space and gave us permission to listen to their planning, teaching, and thinking. We will be forever grateful for their courage. We would also like to thank the teachers, ESL coordinators, and preservice teachers who used the materials in professional development and course settings so that we could improve the content and relevance for teachers with different levels of experience in different contexts. Maggie Hackett and Tabetha Finchum provided a careful review of the materials from the perspectives of teachers and professional development facilitators. As you can see, the development of this book would not have been possible without this incredible team.

Finally, we are grateful for our family and friends who love and encourage us, especially when we escape to write.
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Dr. Kathryn B. Chval is a professor of mathematics education at the University of Missouri. Kathryn’s commitment to educational solutions in mathematics education is rooted in her early experiences teaching elementary grades in underresourced schools in the United States. Kathryn’s research focuses on effective preparation models and support structures for mathematics teachers, effective elementary mathematics teaching for multilingual learners, and curriculum standards and policies. Prior to joining the University of Missouri in 2003, Kathryn was the acting section head for the Teacher Professional Continuum Program in the Division of Elementary, Secondary, and Informal Education at the National Science Foundation (NSF). She worked at the University of Illinois at Chicago from 1989 to 2001 as the codirector on mathematics curriculum development projects and systemic change projects funded by the NSF. She has served as an investigator on projects including the Center for the Study of Mathematics Curriculum, ALL Learn Mathematics, and Collaborative Research: Parents, Teachers, and Multilingual Children Collaborating on Mathematics Together. Additionally, she is the recipient of the prestigious NSF Early Career Award for a project titled A Study of Strategies and Social Processes That Facilitate the Participation of Latino English Language Learners in Elementary Mathematics Classroom Communities. Kathryn’s leadership, research, and service have been recognized with several awards, including the Association of Mathematics Teacher Educators (AMTE) Early Career Award, the INSIGHT Into Diversity Giving Back Award for administrators, the University of Illinois at Chicago College of Education Alumni Award, and the NSF Director’s Award for Program Management Excellence.

Dr. Erin Smith is an assistant professor of mathematics education in the School of Education at the University of Southern Mississippi. Erin received her PhD in learning, teaching, and curriculum from the University of Missouri. Her interest and passion for increasing access to high-quality mathematics for multilingual learners stems from her work teaching mathematics and English. Erin’s research examines the practices of exemplary monolingual teachers of multilingual learners, the preparation of preservice teachers for diverse learners, and the facilitation of parent/guardian collaborations designed to advance children’s success in school mathematics. Prior to joining the University of Southern Mississippi, Erin was an instructor of mathematics at Zayed University in Abu Dhabi, United Arab Emirates, and a teacher of English as a foreign language in Seoul, South Korea. Erin is a member of the National Council of Teachers of Mathematics, TODOS: Mathematics for ALL, and the American Educational Research Association. She is a McNair Scholar; North Carolina State University Building Future Faculty Fellow;
Association of Mathematics Teacher Educators (AMTE) Service, Teaching, and Research (STaR) Fellow; and Service-Learning Fellow at the University of Southern Mississippi.

Dr. Lina Trigos-Carrillo is the chair of the Department of Psychology of Development and Education and an associate professor in the School of Psychology at Universidad de La Sabana in Chía, Colombia. Lina is a professor of literacy education, community education, and qualitative research methods. She received a PhD in learning, teaching, and curriculum from the University of Missouri, and she participated as a research postdoctoral fellow in the project Strengthening Equity and Effectiveness for Teachers of English Learners (SEE-TEL), a National Professional Development Program grant from the U.S. Department of Education. Lina’s research focuses on critical sociocultural perspectives to writing and community/family literacies of people of color and multilingual learners across the Americas. She has conducted qualitative research with multilingual families in the United States and with diverse families and communities in Mexico, Costa Rica, and Colombia. She designs culturally sustaining professional development based on her research experiences in global contexts.

Dr. Rachel J. Pinnow is an associate professor at the University of Missouri in the College of Education. She currently serves as the emphasis area leader of the Teaching English to Speakers of Other Languages (TESOL) program in the Department of Learning, Teaching, and Curriculum. Rachel received her PhD in language and literacy education with a specialization in teaching additional languages (TAL) from the University of Georgia. Rachel grew up speaking multiple languages; taught English in Dalian, People’s Republic of China, for several years; and continues to work with multilingual learners in various educational contexts such as South Africa and many U.S. communities. As a multilingual speaker and an applied linguist, Rachel focuses her research on second language acquisition, classroom interaction, multimodal communication and analysis, social semiotic theory, and positioning theory. Her work seeks to provide insight to how classroom interactions can provide affordances for content and second language learning for multilingual learners. She currently serves as principal investigator on a National Science Foundation–funded grant, Collaborative Research: Parents, Teachers, and Multilingual Children Collaborating on Mathematics Together, that examines positioning among multilingual learners, their family members, and teachers during mathematics education and instruction.
CHAPTER 1
OUR HOPE FOR MULTILINGUAL LEARNERS
TEACHERS WHO INSPIRE

Most of us teachers can recall someone who influenced our entry into education, taught us how to become better teachers, and inspired us to enhance the lives of others. We authors would love to hear the stories about the individuals who inspired you because those are the stories that give us hope. Your decision to read this book tells us you are looking for approaches to enhance the participation and success of multilingual learners. Thank you for making that investment!

Early in my career, I (Kathryn) worked with a variety of elementary teachers as I was writing mathematics curriculum, teaching in K–8 schools, and facilitating professional learning sessions. One of the most instrumental relationships was with a fifth-grade teacher, Ms. Sara Martínez. Ms. Martínez is a bilingual teacher, fluent in English and in Spanish, who is excellent at connecting with both children and their families. She was known as the teacher who held very high expectations for her students. I collaborated with Ms. Martínez to conduct a research study, which involved observing her classroom 60 times during the school year: five times in Week 1, three times a week in Weeks 2 to 6, and one to two times per week for the rest of the school year. I documented a careful record of what happened in the classroom by compiling field notes as well as collecting student work each week in the curriculum materials and samples of writing assignments. A total of 119 mathematics lessons were audio recorded. Ms. Martínez’s classroom was composed of students whose primary language was Spanish. At the time the study took place, Ms. Martínez had a self-contained class of 24 students who represented a wide range of proficiencies in Spanish and English. As with most fifth-grade language learners, the students were still developing in academic English (as opposed to conversational English) proficiency. Figure 1.1 includes the median grade equivalent for the Iowa Test of Basic Skills (ITBS) reading test before entering Ms. Martínez’s classroom compared to the other fifth graders in the school, district, and national norm.

![Figure 1.1 Median Grade Equivalent (Reading) Prior to Entering Ms. Martínez’s Classroom](image)

<table>
<thead>
<tr>
<th>COMPARISON GROUPS</th>
<th>END OF GRADE 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ms. Martínez’s Class</td>
<td>3.7</td>
</tr>
<tr>
<td>Other Fifth Graders in the School</td>
<td>4.0</td>
</tr>
<tr>
<td>Fifth Graders in the District</td>
<td>4.2</td>
</tr>
<tr>
<td>National Norm</td>
<td>4.8</td>
</tr>
</tbody>
</table>
Figure 1.2  Growth in One Year Measured by Median Grade Equivalent (Math Total)

<table>
<thead>
<tr>
<th>COMPARISON GROUPS</th>
<th>END OF GRADE 4</th>
<th>END OF GRADE 5</th>
<th>GAIN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ms. Martínez’s Class</td>
<td>4.3</td>
<td>6.1</td>
<td>1.8</td>
</tr>
<tr>
<td>Other Fifth Graders in the School</td>
<td>4.6</td>
<td>5.8</td>
<td>1.2</td>
</tr>
<tr>
<td>Fifth Graders in the District</td>
<td>4.6</td>
<td>5.6</td>
<td>1.0</td>
</tr>
<tr>
<td>National Norm</td>
<td>4.8</td>
<td>5.8</td>
<td>1.0</td>
</tr>
</tbody>
</table>


Figure 1.2 demonstrates the mathematical gains that Ms. Martínez’s students made in her classroom as measured by the ITBS mathematics assessment. As we can see from the fourth-grade column, the average child in Ms. Martínez’s classroom was half a year behind the expected 4.8. Five of the 24 students (20.8%) performed at the 4.8 level or above. After just eight months in Ms. Martínez’s classroom (fifth-grade column), her students outperformed the other two groups, and 15 of the 24 students (62.5%) performed at the 5.8 level or above. Overall, Ms. Martínez’s students accomplished a great deal in a short amount of time as evidenced by their performance on not only the ITBS, but also other measures collected during the study.

Each day I observed Ms. Martínez’s classroom was memorable, but the first day of the school year particularly stands out. One common misconception about teaching multilingual learners is that they should not be asked to engage in challenging academic work, such as complex mathematics problem solving, until they are at grade level in English language proficiency. Due to this misconception, multilingual learners often languish in academic content classrooms where they continue to fall behind in both academic content learning and second language acquisition as the years of study necessary for grade-level proficiency pass. Since multilingual learners are learning how academic language and academic content work by using these resources to engage in classroom tasks and activities, waiting until they reach grade-level proficiency in English can actually keep them from making progress. The social ramifications are alarming as these learners can come to believe that (because they are not being challenged with grade-level work) they must be deficient in some way that disqualifies them for the same success as their peers.

Fortunately for her students, Ms. Martínez created a challenging and supportive environment for all students where multilingual learners flourished. The emphasis in Ms. Martínez’s classroom was always on solving challenging problems, explaining how to solve hard problems, identifying more efficient ways of solving problems, and investigating more interesting ways of solving problems. When students progressed beyond Ms. Martínez’s own mathematical knowledge, which was very strong, she was not deterred. In order to shape her classroom as a place where learning was valued, she was very open about modeling how learners, herself included, admit when they do not know the
answer or how to proceed in solving a problem. Ms. Martínez made a practice of tackling advanced mathematics problems that she herself struggled with in order to show that “the reason we are in school is we are learning. If we make a mistake, that is great. Let’s put it up on the board so that we can figure out how to fix it.” This approach also made the process of learning more important than producing the product of a correct mathematical answer. How you got to the answer mattered, including where you might have gone awry or faced difficulties in problem solving.

Ms. Martínez also operated in ways that were countercultural in that problem solving was not viewed through an individualized lens—something that occurred inside the learner alone and belonged to the learner alone. Rather, learning was socially distributed, a classroom community process that required that everyone participate, share their knowledge and questions, and share their struggles. By creating a community of learners, students achieved more, not less, as the test scores of her students after one year indicate.

When I showed a video of Ms. Martínez’s teaching at a professional conference, it was clear no one had seen anything like it. I began to wonder: How can I provide opportunities for other teachers to learn about Ms. Martínez’s practice? I will always be grateful for her willingness to open her classroom so that I could share her strategies teaching mathematics to multilingual learners through transcripts of her teaching. To read more about Ms. Martínez’s teaching practice, see Chval (2004, 2012); Chval and Chávez (2011); Chval and Khisty (2009); Khisty and Chval (2002); Morales, Khisty, and Chval (2003); and Razfar, Khisty, and Chval (2011).

**STUDYING TEACHER PRACTICE**

I (Kathryn) designed additional research studies, funded by the National Science Foundation, that involved a variety of elementary teachers so other educators could learn from Ms. Martínez. The research studies that followed involved children wearing video cameras mounted on hats with Velcro® and then more sophisticated wireless video cameras that captured mathematics teaching and learning as shown in Figure 1.3 and Figure 1.4. In the early stages in 2005, Óscar Chávez and I tried out this approach in a first-grade classroom and a fourth-grade classroom to determine if the video cameras would capture useful data.

Then I designed a research study where I collaborated with four third-grade teachers during three academic years from 2009 to 2012. Each week, I met with the teachers to introduce ideas that would influence the design and enactment of instruction, in relation to multilingual learners (i.e., planning sessions). After the planning sessions, the research team (including Rachel Pinnow and Lina Trigos-Carrillo) videotaped two mathematics lessons in each classroom. Then I met with each teacher to debrief lessons and discuss video clips that were filmed during the past week (i.e., debrief sessions). Each session was shaped as a conversation about teaching and learning, rather than a
Our Hope for Multilingual Learners

directive of how the teachers should teach. After each planning session, the teachers would reflect on the conversation, design their own lessons, and create lesson materials. The professional development involved in this process included all the components that Garet and colleagues (2001) identified as critical for effective professional development. For example,

- I worked with the teachers consistently for three years;
- I focused on the content I wanted the teachers to learn—teaching mathematics to multilingual learners in elementary classrooms;
- I integrated lesson planning to connect the work with teachers’ daily experiences and constraints; and
- I integrated discussions and lesson planning so I could assess teachers’ prior knowledge and experiences as I thought about what kinds of questions to pose and in what ways I could facilitate their thinking about teaching multilingual learners, especially through the selection of videos filmed by students in their classrooms.

One third-grade teacher, Laura McKinney, reflected on how the use of the student cameras helped her grow during the first year:

“
The first time I watched a video filmed with the head cameras, I was shocked. I couldn’t believe the things I missed even though I was right there! It concerned me at first, but as the year went on, I realized some great things were happening. I was able to see student interaction without the students feeling the need to please me, because I wasn’t hovering over them. Another benefit [of the video cameras] is the opportunity to see student weaknesses. When students take 10 minutes to start an activity, I know they are struggling
somewhere. I can also see what exactly the students [multilingual learners] are doing in the process of working on a problem. I can see their mistakes as they make them and am better able to understand why they make those mistakes.

The data generated from the student-worn cameras and discussions with teachers provided insight to teachers’ and multilingual learners’ experiences. Throughout this book, you will read transcripts from these interactions as well as from the mathematics lessons that they taught. We use pseudonyms when we reference teachers and students in the book. As we worked with teachers and analyzed the data, we noticed that children in classrooms of teachers who teach like Ms. Martínez also learn to value every person, all the languages they speak, and what they contribute to the classroom community. We are so grateful for the teachers who were willing to invest time to learn how to more effectively teach mathematics to multilingual learners; open their classrooms to multiple video cameras; and, most importantly, share their practice with other teachers. See Figure 1.5 for more information on the classrooms studied.

### Figure 1.5 Teachers Involved in the Studies

<table>
<thead>
<tr>
<th>TEACHER</th>
<th>YEARS OF EXPERIENCE</th>
<th>GRADE</th>
<th>SCHOOL</th>
<th># OF STUDENTS</th>
<th># OF MLLS</th>
<th>LANGUAGES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sara Martínez</td>
<td>20</td>
<td>5</td>
<td>Large urban district</td>
<td>24</td>
<td>24</td>
<td>Spanish</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>96.8% low-income</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>96.9% Hispanic</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>46% limited English proficient</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>21.5% mobility rate</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Courtney Bristow</td>
<td>2</td>
<td>3</td>
<td>Small city</td>
<td>22</td>
<td>3</td>
<td>Spanish</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6.6% Hispanic (year 1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>9.5% Hispanic (year 2)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>58.1% free and reduced-price lunch</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kari Reams</td>
<td>2</td>
<td>1</td>
<td>Small city</td>
<td>22</td>
<td>7</td>
<td>Spanish, Korean, Chinese</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3% Hispanic</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>8% Asian</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>16.9% free and reduced-price lunch</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Roger Jones</td>
<td>15</td>
<td>4</td>
<td>Small city</td>
<td>18</td>
<td>1</td>
<td>Spanish</td>
</tr>
</tbody>
</table>
### 1. Our Hope for Multilingual Learners

<table>
<thead>
<tr>
<th>TEACHER</th>
<th>YEARS OF EXPERIENCE</th>
<th>GRADE</th>
<th>SCHOOL</th>
<th># OF STUDENTS</th>
<th># OF MLLS</th>
<th>LANGUAGES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laura McKinney</td>
<td>1</td>
<td>3</td>
<td>Rural industrial</td>
<td>20</td>
<td>4</td>
<td>Spanish</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>22% Hispanic</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>76% free and reduced-price lunch</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jessica Barnes</td>
<td>3</td>
<td>3</td>
<td>Rural industrial</td>
<td>21</td>
<td>3</td>
<td>Spanish, Russian</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>22% Hispanic</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>76% free and reduced-price lunch</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cindy Keller</td>
<td>18</td>
<td>3</td>
<td>Rural industrial</td>
<td>21</td>
<td>9</td>
<td>Spanish, Russian</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>22% Hispanic</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>76% free and reduced-price lunch</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note: MLLs = multilingual learners.*

### Interacting with Multilingual Learners and Their Families

During the three-year study involving Ms. Bristow, Ms. Keller, Ms. McKinney, and Ms. Barnes, we also interviewed the multilingual learners and their parents. We were well aware of misconceptions about children whose first language is not English among preservice teachers (see Chval & Pinnow [2010] and Vomvoridi-Ivanovic & Chval [2014] for examples). In addition, Ogbu and Simons’s (1998) argument that “the treatment of the minorities in the wider society is reflected in their treatment in education” (p. 161) suggested that we needed to listen to the stories of the families involved. In an effort to provide counter-stories to the deficit-oriented comments we had heard from some educators, we interviewed parents to learn more about the families of multilingual learners. We heard the incredible love that the families had for their children and that they would give up everything—would leave their homeland to come to the United States—so that their children could have what they hoped would be a better life. During these conversations with families, many truths were made evident. As noted, we are aware that misconceptions exist regarding multilingual families. To dispel some of those misconceptions, we created Figure 1.6. We are eager to hear from you what you would add to the list.
Parents make very difficult choices for important reasons that are often unknown to outsiders. You will also have opportunities to learn from parents as you engage in the content of this book. Thank you for participating in these important conversations with us. We know they will influence your teaching of mathematics, the participation of multilingual learners in your classroom, and your engagement with families.

**Figure 1.6  Misconceptions About Multilingual Families**

<table>
<thead>
<tr>
<th>MISCONCEPTION</th>
<th>REALITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Families do not care to attend meetings at schools.</td>
<td>Parents may work multiple jobs to provide for their families in the United States and back home. They may not be able to miss work during the school day or evenings to attend school meetings.</td>
</tr>
<tr>
<td>Families are not interested in information disseminated by the school.</td>
<td>Multilingual families may have limited literacy levels in English and may not understand the flyers, emails, and homework assignments.</td>
</tr>
<tr>
<td>Families are not literate.</td>
<td>They may be extremely well educated, but in a different language.</td>
</tr>
<tr>
<td>Families do not value education.</td>
<td>Multilingual families value education so much they are willing to leave their homeland and (sometimes) live in poverty in the United States so that their children can enjoy quality and safe educational experiences.</td>
</tr>
</tbody>
</table>