Directions: Launch the tasks in a whole group to provide opportunities for students to discuss their understanding of the task and suggest strategies to solve. Organize the students in pairs or groups of four to encourage participation. Provide manipulatives, chart paper, and markers.

**Topic**

Build a polynomial function that models a relationship between two quantities.

**Task**

A local landscaper builds small pools for community centers in an urban area. His standard pool fills a rectangular piece of land and is 10 feet wide and 3 feet deep at the shallowest end. The shallow end is generally 5 feet long to provide enough shallow space for children. The pools he builds have floors that slope steadily from end of the shallow area to the deepest point at the opposite end of the pool and are customizable for the needs of the community. Each additional foot in length he adds to a pool allows him to add one quarter of a foot in depth, based on the slope of the deep end floor. How much water is necessary to build any given pool?

**Facilitate**

Prompt students to create a three-dimensional diagram and attend to precision while labeling their diagram. A possible extension may be to figure out the amount of concrete necessary if pool walls must be 8 inches thick and the floor must be 6 inches thick.

**Make the Math Visible**

Select groups to share a variety of diagramming and problem-solving strategies. Sequence presentations in a way that allows students to build conceptual understanding of how to build a polynomial function from a product of its linear factors.

**Notes**

Instructional mathematics tasks are accessible to all learners because they invite students to wrestle with a problem. Students share their ideas, ask questions of one another, use and apply multiple representations, and collaborate to develop various solution pathways. Then, teachers use students’ solutions to make the math visible, connect prior learning, and forecast new mathematical learning.
2 Topic

Understand solving rational equations as a process of reasoning and explain the reasoning.

The swim course at a sprint triathlon requires athletes to swim out to a buoy that is 400 meters away from the shore and back to the start, where the athletes all transition to the cycling portion of the race. A novice triathlete competing in her first race takes 20 minutes of steady swimming to complete this section, with a 5 centimeters per second current working against her for the first half of the swim and with her during the second half. What would the average speed of the athlete during the swim portion of the race have been if she had been swimming in still waters?

Facilitate

Build contextual understanding by discussing, as a class, the events of a triathlon and the effects of a current on the swim portion of the race. Prompt students to use their understanding of the relationship between distance, rate, and time to make sense of the problem.

Make the Math Visible

Discuss how students used their understanding of the relationship between distance, rate, and time to set up a rational equation to model the problem. Consider examining the graphical representation of the function that relates the time it would take to swim each 400 meters based on a swimmer’s rate in still waters, discussing the meaning of the vertical asymptote in context.

3 Topic

Construct exponential models and use them to solve problems.

Acetaminophen overdose is the leading cause of poisoning and liver failure in the United States. A nurse wants to administer a pain reliever to a patient with liver disease, but the patient has just taken 1,000 milligrams of acetaminophen, which can have a half-life of up to 4 hours in people with damaged livers. The nurse will feel comfortable administering the appropriate medication once the acetaminophen level in the patient’s system drops below 300 milligrams. When is the earliest the nurse can relieve his patient’s pain with the medicine?

Facilitate

Encourage students to use multiple representations to support their conclusions. Select and sequence student responses in a way that builds understanding between the numerical, pictorial, and symbolic representations of the scenario.

Make the Math Visible

Select students to present their strategies in a way that builds a conceptual understanding for half-life as an exponential decay function through its numerical, graphical, and symbolic representations. Make connections between solving by a graphical estimation, by using technology to evaluate a system of constant and exponential functions, and by using logarithms. Compare and contrast these strategies, discussing reasonability within the context of the situation.
Solve systems of nonlinear equations.

Consider the system: \[
\begin{cases}
    f(x) = x(x - 3)(x + 2) \\
    g(x) = x^3 + kx^2 - 2x + 1
\end{cases}
\]
where \( k \) is a constant.

For what values of \( k \) will the system have no real solutions? Multiple real solutions? One unique, real solution?

Prompt students to make use of the structure of the system in order to arrive at a numerical answer. Have students use technology to refer to a graphical representation of the system for reasonability.

Make connections between the graphical representation of the system and the number of real solutions it has for different values of \( k \). Show how an answer can be derived using algebraic methods. Consider having students sketch a system of cubic functions with three solutions and/or describe what would have to be true about the equations in the system for this to occur.
Adapt-a-Mathematical TASK Tool

Do you have a task that is not quite right? Use this guide to adapt the task to meet your needs!

How does the task meet your STUDENTS’ needs?

ACCESS and EQUITY: Ensure that the task is “responsive to students’ backgrounds, experiences, cultural perspectives, traditions, and knowledge” (NCTM, 2014, para. 1, https://www.nctm.org/uploadedFiles/Standards_and_Positions/Position_Statements/Access_and_Equity.pdf). Consider students’ language readiness, including access to mathematical vocabulary.

• How can you differentiate the context of the task to support the students’ backgrounds, experiences, and cultural needs?
• How can you group students to engage the students’ socio-emotional and developmental needs?
• How can you “open up” the task to encourage access to the task for all learners?
• How can you connect the task to the mathematics the students have learned and students’ interests?

How do you PLAN for students to learn from the task?

MATHEMATICAL GOAL: The task should provide students opportunities to access new mathematical knowledge and to solidify, consolidate, or extend knowledge. Tasks can be changed to highlight multiple learning needs and content standards. Ensure that you strategically connect the learning goal to the task.

• What do your students know how to do right now?
• What do you expect your students to understand as a result of this task?
• What do you anticipate students will do? What changes might you make as a result of your anticipation?

FACILITATE: Task facilitation is critical to student success. Consider how you will organize students and design purposeful questions to help them discover and connect mathematics concepts and procedures.

• What questions are you going to ask? What tools will you provide? How will students be grouped?
• How and when will you provide opportunities for student discourse?

How do you move learning FORWARD?

FORMATIVE ASSESSMENT: Collecting information about student understanding will help you adjust instruction as you conduct the task.

• How will you listen, observe, and identify students’ strategies?
• How will you respond to students’ understanding?
• How will you provide feedback to students?
• How will you provide opportunities for students to provide feedback to one another?
• How will you provide opportunities for students to persevere and productively struggle through problems?
• How will you make the mathematics visible for your students?