Task 8.8

Covered With Paint

Mathematics Focus

- Students create functions to model patterns found in geometric and table representations and identify rate of change (slope).

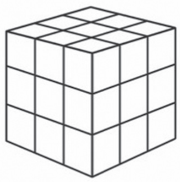
Mathematics Content Standard(s)

- 8.F.4: Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
- A-CED-4: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
- F-IF-4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- F-LE-1: Distinguish between situations that can be modeled with linear functions and with exponential functions.

TASK

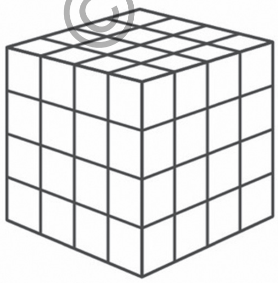
Covered With Paint

1. Selena made a foam cube that measured 3 inches on an edge. She dipped it into paint to completely cover it. When the paint dried, she cut the large cube into 1-inch cubes.



   a. How many 1-inch cubes were in the large cube?
   b. Of the 1-inch cubes, how many had *only* 1 face painted?
   c. How many had *only* 2 faces painted?
   d. How many had *only* 3 faces painted?

2. Devon made a foam cube that measured 4 inches on an edge. He dipped it into paint to completely cover it. When the paint dried, he cut the large cube into 1-inch cubes.



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   b. Of the 1-inch cubes, how many had *only* 1 face painted?
   c. How many had *only* 2 faces painted?
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- F-BF-1: Write a function that describes a relationship between two quantities.

Mathematical Practice(s)

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and express regularity in repeated reasoning.

Vocabulary

- cube
- face
- edge
- vertex
- rate of change
- coefficient

Materials

- 1 Covered With Paint task per student
- At least 27 wood or connecting cubes per group of 4 students (64 cubes is optimal)

3. Record your information in each of the following tables. Build additional cubes as you need to help you see patterns.

Table 1

Number of cubes on one edge (x)	Total number of 1-inch cubes (y)
2	8
3	27
4	
5	
6	
x	

In Table 1, as x increases by 1, what do you notice about the values of y?

Table 2

Number of cubes on one edge (x)	Number of 1-inch cubes painted on 3 faces (y)
2	
3	
4	
5	
6	
x	

In Table 2, as x increases by 1, what do you notice about the values of y?

Table 3

Number of cubes on one edge (x)	Number of 1-inch cubes painted on 2 faces (y)
2	
3	
4	
5	
6	
x	

In Table 3, as x increases by 1, what do you notice about the values of y?

Table 4

Number of cubes on one edge (x)	Number of 1-inch cubes painted on 1 face (y)
2	
3	
4	
5	
6	
x	

In Table 4, as x increases by 1, what do you notice about the values of y?

Task Type

X	Conceptual
	Procedural
	Problem-Solving Application
X	Problem-Solving Critical Thinking

	Reversibility
X	Flexibility
X	Generalization

TASK PREPARATION CONSIDERATIONS

- Can students identify linear and nonlinear patterns in a table?
- How much experience have students had with three-dimensional shapes—in particular, building a cube?
- Will students be able to visualize the cubes that are not on the faces of the larger cube?

SCAFFOLDING OR DIFFERENTIATING THE TASK

- Remind students to build the cubes with their concrete materials.
- Demonstrate how to count the number of painted cubes on a face by using a think-aloud approach or providing different-colored stickers to place on the smaller cube faces.
- Have students focus on the linear relationships before moving to the nonlinear relationships.

WATCH-FORS!

- Students may have inefficient counting strategies.
- Students may forget that there are smaller cubes "inside" the larger cube.
- Students may think that all functions are linear.
- Students may focus on relationships but not use covariational thinking.

EXTEND THE TASK

- Have students graph the relationships in each table and note the similarities and differences of the graphs as they compare to the values in the tables and geometric models.

LAUNCH

1. Place students in groups of 3 or 4.
2. Use wooden or connecting cubes to create a 2 × 2 × 2 cube.
   a. Have students identify the number of faces, edges, and vertices.
   b. Demonstrate dipping the cube into paint.
   c. Ask, "How many of the smaller cubes will have 0 faces painted? 1 face painted? 2 faces painted? 3 faces painted? 4 faces painted? 5 faces painted? 6 faces painted?"
   d. Record students' responses for them to refer to as they work through the task.
3. Build a 3 × 3 × 3 cube and follow the same pattern of questioning.
   a. Record the responses for students to refer to as they work through the task.
4. Distribute the Covered With Paint task.
5. Allow about 25–30 minutes for students to work.

FACILITATE

1. Monitor the groups as they work.
2. Select a group to share their answers to problems 1 and 2.
3. Select a group to share their table entries to verify the data recorded.
   a. Does everyone agree with the entries?
   b. Continue with each table and a different group sharing.

4. Discuss the relationships as x increases by 1.

   a. Focus on the covariational aspects of the relationships using the language "As x increases by 1, y . . ."

EXPECTED SOLUTIONS

1. a. 27 cubes

   b. 6 cubes

   c. 12 cubes

   d. 8 cubes

2. a. 64 cubes

   b. 24 cubes

   c. 24 cubes

   d. 8 cubes

3. Table 1

Number of cubes on one edge (x)	Total number of 1-inch cubes (y)
2	8
3	27
4	64
5	125
6	216
x	x^3

Table 2

Number of cubes on one edge (x)	Number of 1-inch cubes painted on 3 faces (y)
2	8
3	8
4	8
5	8
6	8
x	8

Table 3

Number of cubes on one edge (x)	Number of 1-inch cubes painted on 2 faces (y)
2	0
3	12
4	24
5	36
6	48
x	$12(x - 2)$

Table 4

Number of cubes on one edge (x)	Number of 1-inch cubes painted on 1 face (y)
2	0
3	6
4	24
5	54
6	96
x	$6(x - 2)^2$

CLOSE AND GENERALIZATIONS

1. As time allows, use these questions to extend the task discussion.

   a. If the entries in the tables were graphed as ordered pairs, what could you predict about the shape of the graphs? (Answers will vary.) What would it mean if a curve were drawn through the points?

   b. What quadrant(s) are appropriate for the graphs? (Only Quadrant I.) Why?

   c. Which of the tables represent functions? (All tables.)

   d. Which tables represent a linear relationship? (Tables 2 and 3.) Why?

   e. What did you notice about the rate of change in Tables 2 and 3 as compared to Tables 1 and 4? (Answers will vary. The rates of change in Tables 2 and 3 are each consistent, and the rates of change in Tables 1 and 4 are not.)

   f. Discuss how the rate of change is determined. Write the rate of change as a ratio. (For Table 2, the rate of change, or ratio, is 0:1; for Table 3, it is 12:1 [change in y as compared to change in x].)

   g. Have students graph the points in Tables 2 and 3 on coordinate grids. Use a separate grid for each table.

h. Focus on Table 3. Select two points on the line. Demonstrate how to find the change in x and the change in y from these two points. Write it as a ratio. Simplify the ratio to show it is the same ratio as they found on the table.

i. Have students select two different points. Ask them to find the rate of change in a similar way. Write it as a ratio and simplify it. What do they notice? They should notice that the rate of change is the same no matter what two points are selected.

j. Look at the last row of Tables 2 and 3. Write the equation that represents the relationship. (For Table 2, $y = 0x + 8$, and for Table 3, $y = 12x - 24$.) What do students notice about the equations and the rate of change? (The rate of change is the coefficient of x.)

k. What relationship do students notice between the size of the coefficient of x and the steepness of the graph?

POST-TASK NOTES: REFLECTIONS AND NEXT STEPS

- Is students' spatial sense well developed, or is this an opportunity for learning?
- Can this task be used in later lessons to connect linear functions with polynomial functions?
- Did the task provide enough support for students to recognize that the slope of a line is the same no matter which two points are chosen?
- Could this task be extended by using a different three-dimensional shape?

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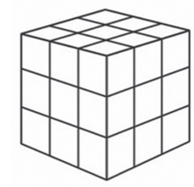

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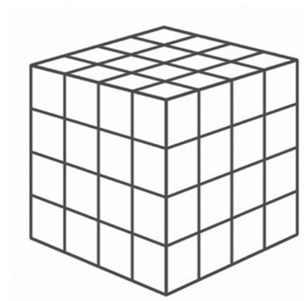

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