## FIGURING OUT IN MATHEMATICS

Teaching and Learning

Moving Beyond Basic Facts and Memorization

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## Thank you

 FOR YOUR INTEREST IN CORWINDifferent types of practice engage students in different ways. Some spark engagement, interaction, and discussion. Other practices are opportunities to work independently. Try this routine with your students to help them develop flexibility and efficiency, then see what other games, routines, an focus tasks Figuring Out Fluency has to offer!

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## TYPES OF FLUENCY PRACTICE

Different types of practice engage students in different ways. Some spark engagement, interaction, and discussion. Other practices are opportunities to work independently. Work within different types of practice nurtures ownership of a strategy through transferring its application from one setting to another. The upcoming practice types are not necessarily the entire list of practice types. Rather, they are the types or structures of practice that we feature throughout this series. They include fluency routines, worked examples, games, and centers-each of which is discussed in this section..

## FLUENCY ROUTINES

Instructional routines have become popular resources for mathematics instruction in recent years (Kelemanik et al., 2016; Parrish, 2014; SanGiovanni, 2020; Shumway, 2011; Zwiers et al., 2017). A routine is a familiar, adaptable protocol for engaging students in learning through thinking and discussion. Some teachers equate routines with Number Talks, but others see Number Talks as a specific routine with a prescribed (though flexible) protocol. Routines are an essential part of mathematics classrooms, providing structure as they establish expectations for engagement and participation. Routines support management and foster positive mathematics relationships within the classroom community (Berry, 2018). Importantly, different routines serve different learning goals. All the routines in this book, for example, focus on procedural fluency-attending to any or a combination of the three components of fluency and the six Fluency Actions. A fluency routine can be naturally incorporated into a series or set of other number routines you currently use.

The exchange of ideas during a routine is essential for advancing student fluency. Verbalizing one's thinking deepens learning (Bartel, 2016). It causes students to break down information into bits and to make sense of each bit and connect it to other bits. The brain organizes and reorganizes as a process and understanding check. It also provides opportunity for identifying the gaps in one's process or logic. The listener has to take in new information and contrast it with their own. They have the opportunity to reassess their own strategy and change it or affirm their strategy and share it.

The "Or You Could ..." routine is designed to prompt students to develop flexibility, as different problems on the list can be decomposed in various ways, lending to a number of strategies. It also focuses on efficiency as students evaluate which strategy makes sense given the numbers in the problem.

## ACTIVITY 6.2 ROUTINE: "OR YOU COULD . . ."

Materials: None required
Directions: Pose a few expressions, one at a time. Direct students to think about another way to write the expression (not solve it). Students only find an alternative expression that will be useful in solving the problem efficiently. Then, use a think-pair-share process: For each expression, students think of one way to think of the numbers differently in order to multiply. Next, they pair and compare their rethinking of the problem. Then, the teacher starts a "We can ..." list on posterboard or a whiteboard, as students share a way they can rethink the problem.

For example, with these equations- $19 \times 3,35 \times 4$, and $6 \times 17-$ sharing sounds like this: "You could think of $19 \times 3$ as $10 \times 3+9 \times 3$." Another student says, "Or you could do $20 \times 3-3$." Another shares, "You could do $20+20+20-3$." And so on. After ideas are posted on the "We can ..." list, the teacher asks, "Which of these options do you like and why?" Again, think-pair-share. Repeat the process with other examples.

## TEACHING TAKEAWAY

Learners build confidence when they see that their strategy is reasonable and taken up by others. This shapes their mathematics identity and forges their mathematical agency.

Discussion within routines reassures students that their emerging, possibly less orthodox strategies are viable and are actually used by others. Learners build confidence when they see that their strategy is reasonable and taken up by others. This shapes their mathematics identity and forges their mathematical agency.

On the other side of getting students to take up and use a strategy is having them overuse a strategy. Consider a classroom where fluency practice has focused on Use Partials as a strategy. The teacher notices that students solely rely on the strategy, even though other strategies have been learned. She decides to use the "Why Not?" routine described in Activity 6.3.

## ACTIVITY 6.3 ROUTINE: "WHY NOT?"

Materials: None required
Directions: Provide a few expressions. Prompt students to identify a strategy that would not be the most efficient for that expression-for example, you might use $8 \times 25,39 \times 6$, and $3 \times 48$.

Ask students to tell a strategy they would not use. Then say, "Why not?" Students share their rationale, and other students can agree or disagree if that is a strategy that is not a good fit for the problem (and why not).

Imagine that the example in Activity 6.3 is playing out in a fifth-grade classroom. For the first problem, $8 \times 25$, students pause to think of how they would solve it. Break Apart to Multiply is one good option. The Halve and Double strategy to create $4 \times 50$ (or again for $2 \times 100$ ) is a good option. Recognizing 25 s and knowing 8 is 200 is another efficient option. So a student might offer the standard algorithm for the strategy they would not use. And when asked "Why not?", they might say, "Because I just know that four 25 s is 100 , so 8 is 200 ." Or a student might name "Use Partials" for the strategy they would not use for $39 \times 6$, and their "why not" might be that they would rather do the problem in their head (and for Use Partials, they would need to write down the actual partial products). The "why not" is based on the numbers in the problem and the students' preferences.

Different fluency routines focus on different elements of fluency. These two routines focus on selecting and adapting appropriate strategies. We have included other fluency routines throughout the book, which can be easily found by looking for the activities labeled as routines in the Activity List in Appendix A.

## Appendix A

## Activity List

This at-a-glance list includes all the activities in the books. Activities are labeled as either games, routines, or focus tasks.

|  | ACTIVITY | TYPE | FOCUS | PAGE |
| :---: | :---: | :---: | :---: | :---: |
|  | Just Right | Game | Students match a problem to an appropriate strategy. | 4 |
|  | Strategories | Game | Students generate example problems to "fit" strategies. | 8 |
| 1.3 | "Islt Reasonable?" | Routine | Students critique estimated problems to determine if they are reasonable or not. | 12 |
|  | Good Choice or Bad Choice | Focus Task | Students review a worked example and decide if the strategy used was a good choice or not. | 14 |
|  | "That One" | Routine | Students look at a series of problems and select the ones that would be solved most efficiently with a standard algorithm. | 30 |
|  | Compare and Declare | Focus Task | Students explore subtraction problems, deciding whether a "take away" or "find the difference" interpretation is more efficient. | 39 |
|  | Break It to <br> Make It: Area <br> Model | Focus Task | Students find as many ways as they can to break apart a factor to find the product. | 50 |
|  | Distance to 10 <br> Walk | Focus Task | With a number path or number line on the floor, students stand on a given number (e.g., 8 or 13) and count steps to see how far they are from 10. | 53 |
| 3.3 | Five Ways, Most Ways | Focus Task | Individually, students brainstorm ways to break apart a number; then, they work with a team to share and compare options. | 54 |
|  | Think 10 and Back Again | Focus Task | Students explore $\times 9$ concretely by building stacks of $\times 10$ and visualizing stacks of $\times 9$. | 61 |

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|  | ACTIVITY | TYPE | FOCUS | PAGE |
| :---: | :---: | :---: | :---: | :---: |
| 3.5 | "Paired Quick Looks" | Routine | A known fact is paired with a derived fact strategy so that students can visualize the relationship. | 62 |
| 3.6 | Fill the Chart | Game | A known fact is paired with a derived fact strategy so that students can compare the relationship in a chart. | 63 |
| 3.7 | Stay or Go | Game | Students practice front-end estimation using a Bottom-Up Hundred Chart. | 69 |
| 3.8 | "Between and About" | Routine | Students practice finding the range within which the answer lies and then also select a strategy to estimate. | 72 |
| 4.1 | What's the Temperature? | Focus Task | Students work with vertical number lines in context to develop and reinforce strategies. | 80 |
|  | "Strategize Your Strategy" | Routine | Students focus on identifying when a Break Apart strategy is the best method. | 91 |
| 4.3 | Show Me! | Focus Task | Students are exposed to a strategy and asked to generalize it by proving that is always works. | 99 |
| 5.1 | Combinations | Game | Students make combinations of numbers with the goal of eliminating all of the cards in their hand. | 113 |
| 5.2 | Make It, Take lt | Game | Students work with combinations of numbers (such as $100,1.00$, etc.). Players decompose to find combinations and remove their pieces. | 115 |
| 5.3 | Race to 1,000 | Game | Students practice working with counting by/grouping 25s. | 117 |
| 5.4 | "25s Count" | Routine | Students practice counting and estimating with 25 s to develop automaticity with 25 s. | 118 |
| 5.5 | 15 sand 30 s | Game | Students play a game to build automaticity with $15 \mathrm{~s}(15,30,45,60,75$, etc.). | 119 |
| 5.6 | ForKeeps | Game | This game practices doubling numbers. Students decide to keep the double of a generated roll or to get rid of it. The kept doubles add up to create a score. | 120 |
|  | The Splits | Game | This game practices halving numbers. The goal is to place halves in order from least to greatest in the fewest amount of turns. | 121 |

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(Continued)

|  | ACTIVITY | TYPE | FOCUS | PAGE |
| :---: | :---: | :---: | :---: | :---: |
| 5.8 | "AString of Halves" | Routine | Students practice halving numbers and learning how to generate halves for more complicated numbers. | 122 |
| 5.9 | Clearthe Deck | Game | Students play independently to make equivalent fractions using all of the cards in their deck. | 124 |
| 5.10 | Conversion Fish | Game | Students use cards of percentages, decimals, and fractions to make sets of conversions ( $\frac{1}{2}, 50 \%, 0.5$ ) with the goal of removing all of their cards. | 126 |
|  | Strategy <br> Problem Sort | Focus Task | Astrategy is identified. Students sort problems into groups that fit that strategy and problems that don't fit that strategy. <br> Fitting for independent practice or a center. | 132 |
| 6.2 | "OrYou <br> Could ..." | Routine | Students work to rename problems to express more efficient ways of thinking about them. | 140 |
| 6.3 | "Why Not?" | Routine | Students share their thinking about the practicality of a given strategy for a given problem. | 141 |
|  | "The Best Tool" | Routine | Students discuss the best tool for solving a problem (mental, written, calculator). | 143 |
| 6.5 | "Share- <br> Share- <br> Compare" | Routine | Students exchange and compare their strategies for solving problems in this engaging activity. | 146 |
|  | A Winning Streak | Game | Students find sums and cover them on a game board with the goal of creating as many lines of three in a row as possible. | 148 |
| 6.7 | The Make 100 Learning Center | Focus Task | Students arrange digits to make target sums with two-digit numbers. Fitting for independent practice or a center. | 151 |
| 7.1 | Sum War | Game | In this traditional game, students use sums to collect cards from an opponent. | 158 |
|  | "Strategize First Steps" | Routine | Students identify and share their first steps for working with a problem. | 171 |

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