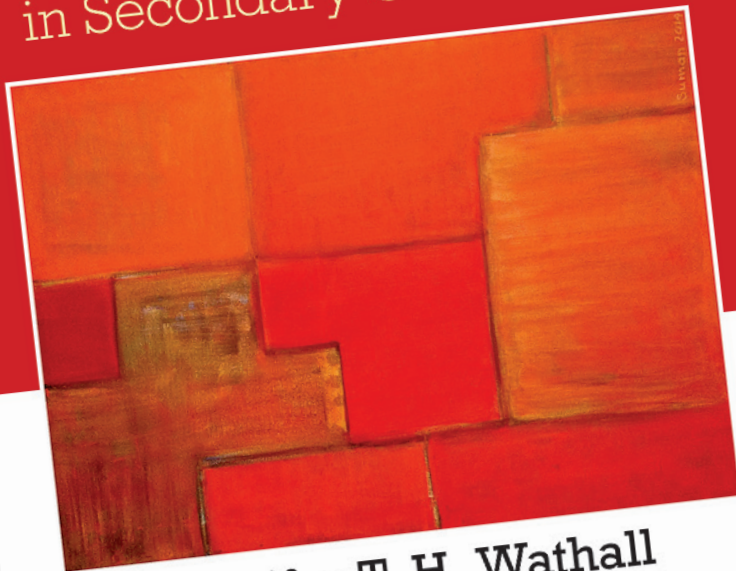


Concept-Based MATHEMATICS

Teaching for
Deep Understanding
in Secondary Classrooms



Jennifer T. H. Wathall

Foreword by H. Lynn Erickson



Thank you

FOR YOUR
INTEREST IN
CORWIN

Please enjoy this complimentary excerpt from *Concept-Based Mathematics* by Jennifer T.H. Wathall. Learn how to design structured inquiry, guided inquiry, and open inquiry tasks on the topic of straight lines.

LEARN MORE about this title, including Features, Table of Contents, and Reviews.

How Do I Design Inductive, Inquiry-Based Math Tasks?

Main Text: Chapter 5. How Do I Captivate Students? Eight Strategies for Engaging the Hearts and Minds of Students

One of the tenets of concept-based curriculum and instruction is the use of inductive teaching approaches. This means students are given specific numerical examples to work out and are guided to the generalizations. It may be appropriate to use different levels of inquiry—structured, guided, or open—depending on teacher experience and student readiness. The levels of inquiry also provide a differentiation strategy.

In this module you will find the following resources:

- Examples of different levels of inquiry using inductive approaches
- An example of an inductive inquiry task and the use of a hint jar
- A template for planning inductive inquiry tasks
- Discussion questions for Module 5
- An opportunity for reflection

FIGURE M5.1: EXAMPLE OF STRUCTURED INQUIRY TASK: LINEAR EQUATIONS

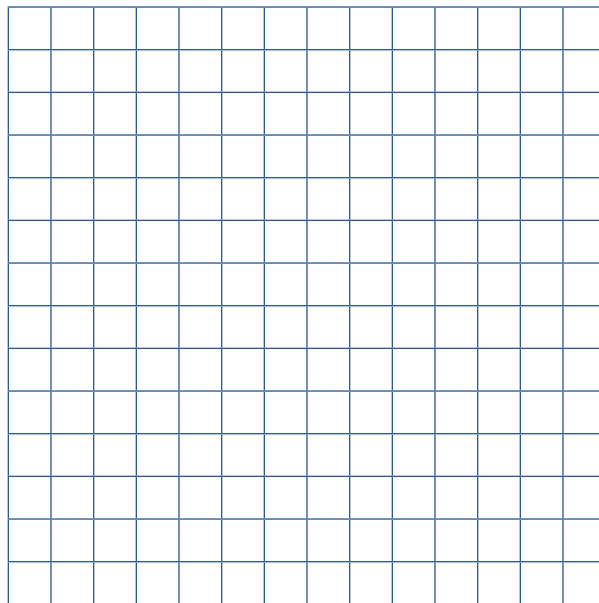
Investigating Straight Lines

Part 1

The cost of a taxi is a flat rate of \$1 and then \$3 for every mile travelled. Complete this table in which y represents the total cost of the taxi ride and x represents miles travelled.

x (miles)	0	1	2	3	4	5	6	7	8
y (total fare)									

Draw the y and x axes below and plot the points from the table.



Can you think of a function that represents the total fare for a taxi ride travelling x miles?

$y =$

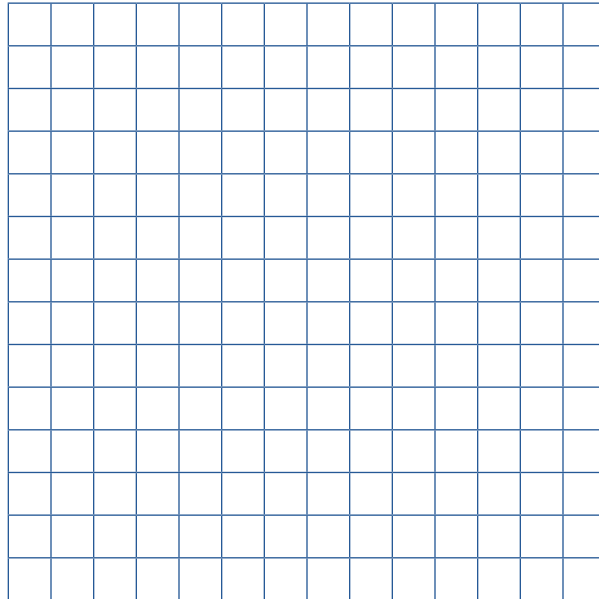
What is the cost per mile? How is this represented on the graph?

Part 2

The cost of a taxi is a flat rate of \$2 and then \$4 for every mile travelled. Complete this table in which y represents the total cost of the taxi ride and x represents miles travelled.

x (miles)	0	1	2	3	4	5	6	7	8
y (total fare)									

Draw the y and x axes below and plot the points from the table.



Can you think of a function that represents the total fare for a taxi ride travelling x miles?

$y =$

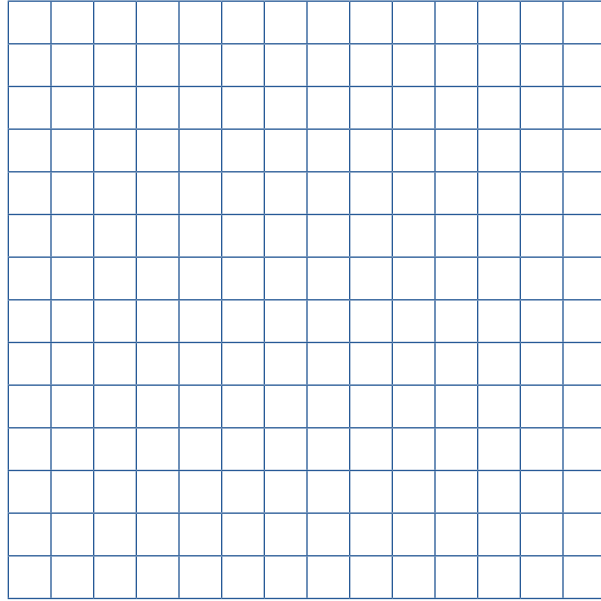
What is the cost per mile? How is this represented on the graph?

Part 3

The cost of a taxi is a flat rate of \$3 and then \$9 for every mile travelled. Complete this table in which y represents the total cost of the taxi ride and x represents miles travelled.

x (miles)	0	1	2	3	4	5	6	7	8
y (total fare)									

Draw the y and x axes below and plot the points from the table.



Can you think of a function that represents the total fare for a taxi ride travelling x miles?

$$y =$$

What is the cost per mile? How is this represented on the graph?

Can you think of a function that represents the total fare for a taxi ride travelling x miles if the flat rate was m and the cost per mile was b ?

$$y =$$

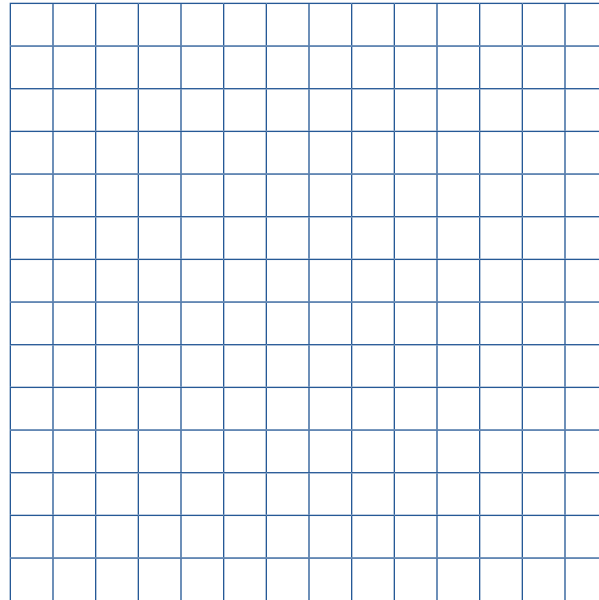
What is the cost per mile? How is this represented on the graph?

Investigating Straight Lines

Part 1

The cost of a taxi is a flat rate of \$1 and then \$3 for every mile travelled.

Draw the y and x axes below and plot the points that represent the total cost (y) of a taxi ride for different miles (x) travelled.



Can you think of a function that represents the total fare for a taxi ride travelling x miles?

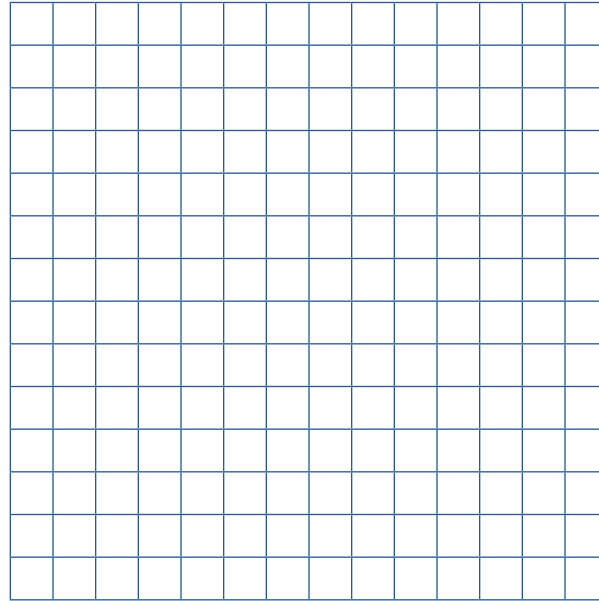
$y =$

What is the cost per mile? How is this represented on the graph?

Part 2

The cost of a taxi is a flat rate of \$2 and then \$4 for every mile travelled.

Draw the y and x axes below and plot the points that represent the total cost (y) of a taxi ride for different miles (x) travelled.



Can you think of a function that represents the total fare for a taxi ride travelling x miles?

$$y =$$

What is the cost per mile? How is this represented on the graph?

Can you think of a function that represents the total fare for a taxi ride travelling x miles if the flat rate was m and the cost per mile was b ?

$$y =$$

What is the cost per mile? How is this represented on the graph?

FIGURE M5.3: EXAMPLE OF OPEN INQUIRY TASK: LINEAR EQUATIONS

Investigating Straight Lines

Investigate the effects of the parameters m and b on the linear function $y = mx + b$ and explain what happens when m and b take different values. Use real-life examples to illustrate your explanations.

FIGURE M5.4: EXAMPLE OF INDUCTIVE INQUIRY TASK: ADDING AND SUBTRACTING NEGATIVE NUMBERS

Temperature Scales

The lowest temperature recorded in Washington is -26°C .

Mark this on the temperature scale on the right with a **W**.

A heat wave sweeps Washington and takes away this cold temperature, resulting in a temperature of 0°C .

This could be represented as

-26	take away	-26
Or in mathematical symbols		
$-26 - (-26) =$		
Another way to express this heat wave is to add 26		
$-26 + 26 =$		
What conclusion can you make about subtracting a negative number? Provide an example of your own that shows how you take away something that is already negative.		



FIGURE M5.5: EXAMPLE OF INDUCTIVE INQUIRY TASK: USING THE TRI-MIND ACTIVITY

Hint Jar

You owe your mum \$10. This means you have $-\$10$. How do you take this away? Explain and show the sum.

You tell someone either to eat, to not eat, or to **NOT** not eat. What does **NOT** not eat mean?

Good things happen to good people = GOOD (positive)

Good things happen to bad people = BAD (negative)

Bad things happen to good people =

Bad things happen to bad people =

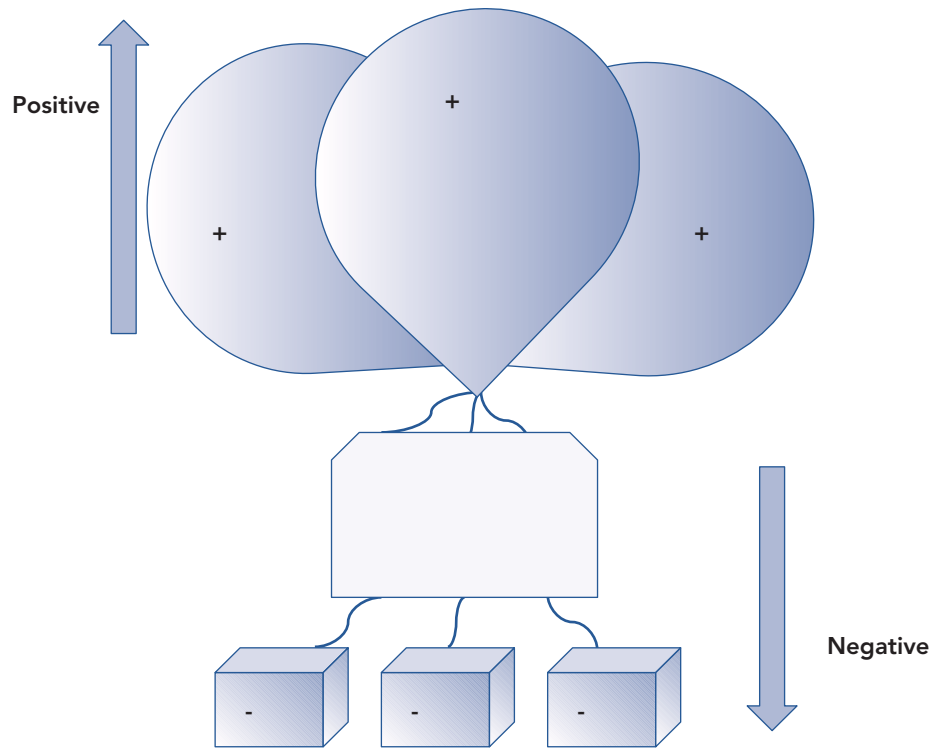
Think of your own analogy and write it here:

FIGURE M5.6: EXAMPLE OF INDUCTIVE INQUIRY TASK: ADDING AND SUBTRACTING POSITIVE AND NEGATIVE NUMBERS

Hot Air Balloons

The hot air balloon basket is floating in the sky with the balloons and weights. Fill in the following table:

Take Away or Add Balloons	Basket Moves up (+) or Down (-)
Add 3 balloons	
Take away 3 balloons	
Take Away or Add Weights	Basket Moves up (+) or Down (-)
Add -3 weights	
Take away -3 weights	
Write your own sums that represent the four operations above:	



What generalizations can you make about adding and subtracting positive and negative numbers? Provide several examples to illustrate your explanations.

FIGURE M5.7: TEMPLATE FOR DESIGNING INDUCTIVE INQUIRY TASKS

Designing Inductive Inquiry Tasks

Generalization:	
------------------------	--

Guiding questions

Factual Questions	
Conceptual Questions	
Debatable Questions	

Inductive Student Task	Prompts
Specific numerical example	
Second specific numerical example	
Third specific numerical example, if appropriate	
Students form generalizations	

Discussion Questions for Module 5

1. What is the difference between inductive and deductive teaching approaches?
2. What are the different levels of inquiry?
3. When and how would you use the different levels of inquiry?
4. How do you use the hint jar for the activity on negative numbers?

An Opportunity for Reflection

Write a headline about inductive inquiry tasks

This thinking routine helps you to summarize the essence or the most important ideas to you regarding inductive inquiry tasks.

FIGURE 5.2: THE DEVELOPING CONCEPT-BASED TEACHER: UNDERSTANDING CONCEPT-BASED CURRICULUM AND INSTRUCTION

Sum of the Roots of Quadratics
Product of the Roots of Quadratics

For the following quadratic functions, find the roots and sketch the function and write down any significant features of these curves. What do the roots of the quadratic function tell us?

1. $y = 3x^2 + 5x$	Sketch
2. $y = x^2 - 5x - 6$	
3. $y = 4x^2 - 4x + 1$	
4. $y = 2x^2 - 13x - 7$	

FIGURE 5.2: (CONTINUED)

Complete this table:

Quadratic function	Roots	Sum of Roots	Product of Roots
1. $y = 3x^2 + 5x$			
2. $y = x^2 - 5x - 6$			
3. $y = 4x^2 - 4x + 1$			
4. $y = 2x^2 - 13x - 7$			

From the table, can you make a generalization about the sum and product of roots for a quadratic function?

Quadratic Function	Roots	Sum of roots	Product of Roots
$y = ax^2 + bx + c$	α and β		

In pairs, write down in words the relationship between the sum of the roots and the coefficients for a quadratic equation. Write down in words the relationship between the product of the roots and the coefficients for a quadratic equation. (Generalizations)

(Continued)

FIGURE 5.2: (CONTINUED)

The Roots of Cubic Functions

Test to see whether your generalization works for cubic functions. You may use specific examples to try your prediction or use an algebraic proof.

Here are some specific cubic functions for you to try if you are not using the proving and reasoning process. Use your graphing software or a graphical display calculator to find the roots and sketch.

1. $y = (x + 5)(x - 6)(x + 7)$	Sketch
2. $y = 2(x + 3)(x - 3)(x + 1)$	
3. $y = 2(x - 1)(x + 2)(x + 1)$	
4. $y = 2x^3 - 5x^2 - 6x + 4$	

FIGURE 5.2: (CONTINUED)

Complete this table:

Cubic function	Roots	Sum of Roots	Product of Roots
1. $y = (x + 5)(x - 6)(x + 7)$			
2. $y = 2(x + 3)(x - 3)(x + 1)$			
3. $y = 2(x - 1)(x + 2)(x + 1)$			
4. $y = 2x^3 - 5x^2 - 6x + 4$			

From the table, can you make a generalization about the sum and product of roots and the coefficients for a cubic function?

Cubic Function	Roots	Sum of Roots	Product of Roots
$y = ax^3 + bx^2 + cx + d$	$\alpha, \beta,$ and γ		

In pairs, write down in words the relationship between the sum of the roots and the coefficients for a cubic equation. Write down in words the relationship between the product of the roots and the coefficients for a cubic equation. (Generalizations)

(Continued)

FIGURE 5.2: (CONTINUED)

The Sum of the Roots and the Product of the Roots of a Polynomial

Quadratic Function	Roots	Sum of Roots	Product of Roots
$y = ax^2 + bx + c$	α , and β		

Cubic Function	Roots	Sum of Roots	Product of Roots	Product Pair and Sum
$y = ax^3 + bx^2 + cx + d$	α , β , and γ			

What is the pattern?

Polynomial Function	Roots	Sum of Roots	Product of Roots

In pairs, write down in words the relationship between the sum of the roots and the coefficients for any polynomial. (Generalizations)