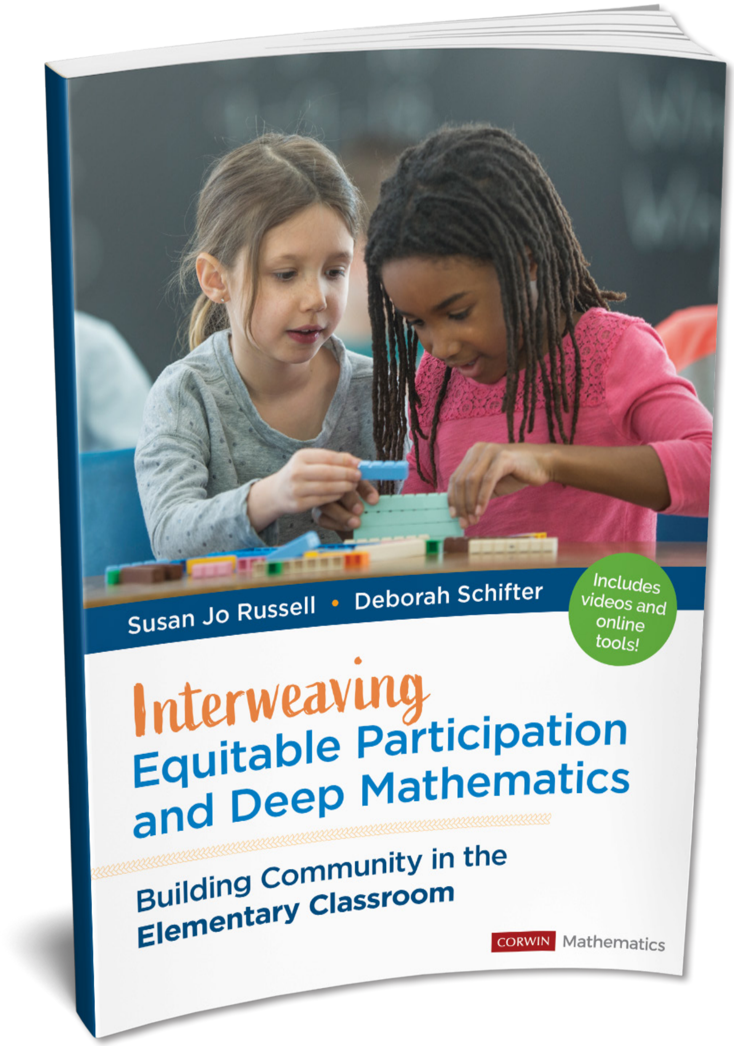

Thank you

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Please enjoy this complimentary excerpt from Interweaving Equitable Participation and Deep Mathematics.

LEARN MORE about this title!

Introduction

What Is a Mathematics Community?

What does a mathematics community look like in an elementary classroom? How do we—teachers, coaches, administrators, all of us who support student learning—engage young mathematicians in deep and challenging mathematical content? How do we ensure that every student contributes a voice to this community, including students who have been historically marginalized in mathematics, students who have not believed they have mathematical ideas that are important to share, or who, when they have tried to express their ideas, have not been heard? These are the core questions this book seeks to address.

This book focuses on the interweaving of two commitments to children: a commitment to teaching deep and challenging mathematics and a commitment to equitable participation for all students in the classroom community. Without the opportunity for students to engage in significant mathematical content, a focus on equity is empty. If there are systems in place to ensure that every student speaks, but the math content is superficial and devoid of sense-making, we are not preparing students to become mathematical thinkers. On the other hand, if we attend exclusively to the rigor and depth of the mathematics, a few students may dominate, and what's perceived to be a correct and complete response from one or two students may stop continued discourse. Without attention to how each student engages with the content, the depth of the mathematics makes no difference for too many students.

In the intersection of deep mathematics and equitable participation, we have classrooms in which the *mathematics* content is significant, and the *community* enables each student to grow in understanding through their participation. In these classrooms, each student is assumed to have mathematical ideas, and it's the work of all of us to learn to listen for them. But as classroom teachers, coaches, instructional leaders, and others responsible for students' learning, how do we build and sustain such a community?

Four Aspects of a Mathematics Community

For two years, we, the authors of this book, undertook a research project in which we visited and videotaped lessons in the classrooms of six public elementary school teachers who were working to create classroom communities in which all students were engaged in serious content. Throughout this book we will refer to them as our Collaborating Teachers. We documented lessons, collected student work and teachers' writing, and reflected on these lessons with the teachers in order to uncover key ideas that characterized their evolving mathematics communities. As you move through the book, you yourself will interact with the videos and teachers' writing, observing and reflecting on students' thinking as well as the actions and thoughts of the teachers as they build mathematical communities in their classrooms. You will also hear the reflections and observations of our Critical Friends, who brought their own experiences to bear on what they saw in these classrooms. We'll explain more about this in a bit.

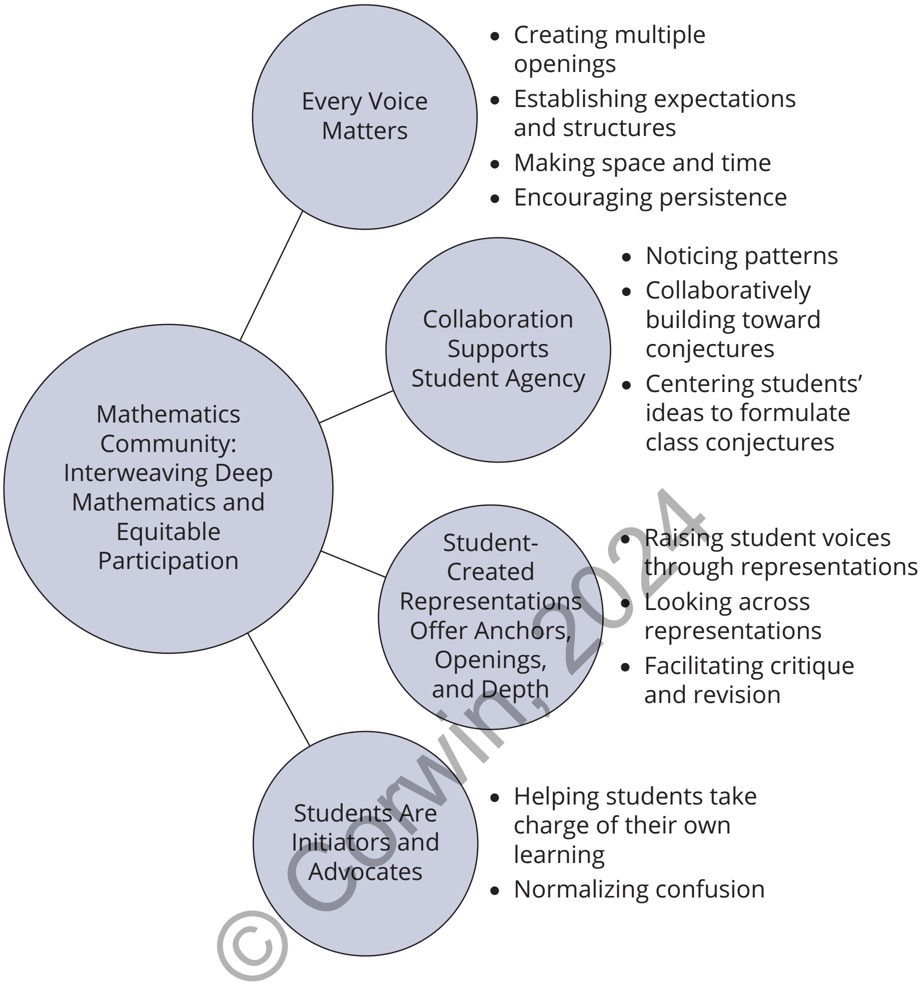
For our Collaborating Teachers, community was characterized by four main ideas: *every voice matters; collaboration supports student agency; student-created representations offer anchors, openings, and depth;* and *students are initiators and advocates for their own learning.* This book is therefore structured in four parts, each focusing on issues related to one of these ideas. While these four aspects of community interact and strengthen each other, we separate them in order to dig more deeply into what it takes to build each facet of a mathematics community (see Figure Intro.1).

Part One: Every voice matters.

In a mathematics community focused on student thinking, teachers establish classrooms in which students learn to take on the responsibility of sharing their ideas and attending to those of their classmates. The chapters in Part One focus on the importance of trusting students to take on challenging ideas as they develop their *identities* as doers of mathematics—how they view their own confidence and competence in approaching mathematical problems and questions. The videos illustrate how students are offered many modes of participation as they are beginning to develop their *mathematical agency*, that is, their inclination and ability to rely on their own reasoning. Both students who are eager to contribute their ideas and students who need more time and support to articulate their thoughts are included in the classroom discourse. Teachers and students listen intently to discern the sense in each student's thinking.

The culture of a mathematics community, as seen in the videos, is established through the teachers' sustained and intentional efforts. In Part One, our collaborating teachers discuss the norms they set and the tools they provide in the beginning of the year to help students understand what it means to contribute to a mathematics discussion. Maintaining that culture requires the efforts of all participants—students and teacher—throughout the year.

Figure Intro.1 • Four Aspects of a Mathematics Community



Part Two: Collaboration supports student agency.

Mathematics is about complex ideas. Even in the primary grades, if given the opportunity and the tools, students engage in deep and abstract mathematical ideas. Unlike the image of the solitary, brilliant student working alone to solve every problem, much of mathematics requires collaboration, or what Aguirre et al. (2024) call "collective mathematical agency": "Classrooms of students can exhibit *collective mathematical agency* when teachers and their students act together to solve problems, working from the shared belief that viable strategies can be developed and solutions can be found. Different students can contribute different elements to this collective agency" (p. 17).

The chapters in Part Two focus on how students build ideas together as they notice patterns and articulate conjectures based on those patterns. The videos and commentary illustrate several related ideas: how offering partially formed or not yet well-articulated ideas for consideration is productive and, in fact, critical to the work of the community; how the practices of questioning and revising contribute to the community's ideas; and how it's everyone's responsibility to try to understand and build on each other's thinking.

Part Three: Student-created representations offer anchors, openings, and depth.

Student-created representations in the form of pictures, diagrams, models, and story contexts ground student ideas, encourage interaction, and draw more students into mathematical thinking. In contrast to the common view that students must be weaned off the use of diagrams and manipulatives to engage in abstract realms, understanding mathematics deepens for all students when they make connections across different forms of representation. Further, creating and explaining their own pictures, models, and story contexts are key parts of students' expressions of their own mathematical identities. You will see in Part Three how students are passionate and engaged as they use their own representations to explain their thinking, field questions from other students, and revise their representations to make their ideas clearer.

The chapters in Part Three present students' representations of pairs of related equations or story problems that illustrate a given generalization. By referring to their representations, students create meaning for symbols. Representations can also be key to mathematical argument, demonstrating *why* a procedure works or *why* a generalization is true.

The videos show students working together to interpret a set of representations chosen by the teacher from those created by the class. Commentary considers such issues as the connections made by looking at different representations, how a teacher selects which representations to share, and the value of choosing some representations that may need revision.

Part Four: Students are initiators and advocates for their own learning.

As students collaborate to build ideas together, they learn to pose their own questions and challenges. Students might ask questions such as the following: Will this pattern work with a different kind of number? What if I try it with very large numbers or very small numbers? Will it work the same for odds and evens? In this way, students become initiators of their own learning.

Students also advocate for themselves by identifying and articulating their confusion. When students engage in discussion with the expectation that they can understand, they learn to pause the discussion if they don't understand. Such pauses are recognized as a contribution to the discourse, allowing all community members to dig deeply and to explain their ideas more clearly. In these ways, students become advocates for their own learning while enhancing the class's collective agency.

Part Four illustrates how each student takes on the responsibility to be an active member of the mathematics community *and* an advocate for their own learning. The chapters include discussion of such themes as strategies a teacher might employ to encourage students' mathematical curiosity while maintaining the coherence of the lesson and what both the teacher and students gain by taking time to address a student's expressed confusion.

The Math

As we examine mathematics community, we give equal weight to "mathematics" and "community." While there are many discussions in the field of education that focus on equitable participation and the development of community, the subject and content of mathematics is sometimes an afterthought at best, or entirely absent, with primary focus put on literacy, history, or science. Is this because classroom mathematics is still seen as the learning of facts and algorithms but not a domain of ideas? Is it because there is still a persistent, perhaps unconscious, belief that some students are wired to do well in mathematics, but others are not?

In the video clips you will be studying, the mathematics students are engaged in centers on the core curriculum content of basic operations—addition, subtraction, multiplication, and division—but it is about formulating and investigating generalizations rather than explicitly learning strategies for solving individual arithmetic problems. The development of accuracy, flexibility, and fluency in solving arithmetic problems is, in itself, an important and fascinating topic. What is also important is for young students to dig even more deeply into the basic operations, through three practices that are highlighted in these videos: noticing patterns and regularities in the number system, conjecturing about what is general in those patterns, and creating a variety of representations to show why and how those patterns hold.

For example, when first graders encounter the operation of addition, they naturally begin to notice and describe patterns. Imagine a group of students generating addition combinations that make 10. A student says: *4 plus 6 makes 10, so 6 plus 4 has to make 10*. Another student chimes in: *I have another one—8 plus 2 is 10, so 2 plus 8 works, too.* And a third student says, *but 5 plus 5 is in the middle so it doesn't have a turn-around.* These students, at the beginning of their mathematical journey into addition, are already starting to notice something that is true about addition *in general*, and they have invented a name, "turn-arounds," for their idea (what they will, in later years, encounter as the commutative property of addition). What if the teacher were to challenge them further to state precisely what they mean by "turn-arounds"? What if the teacher were to ask, *Does this work just for these numbers, or does it work for other numbers, too? How do you know?*

In this book, you will see young students investigate patterns in the number system with intelligence and enthusiasm. When given the opportunity by teachers who show genuine interest in their ideas, students enact their intellectual agency by bringing to these problems their own ways of thinking, and they express their identities as they develop and explain their own models, pictures, and diagrams. The teachers in this book assume the brilliance of their students (Aguirre et al., 2024; Delpit, 2012; Leonard & Martin, 2013; Lewis, 2018; National Council of Supervisors of Mathematics & TODOS, 2021) and support them to develop their intellectual power.

The mathematics content in this book, then, is about the core idea of *generalizing*: finding and proving what holds true across multiple, related examples. The generalizations explored by students in the videos are fundamental to a complete and deep understanding of the operations and connect elementary arithmetic to later study of algebra. Just as important, the mathematical practices that are part of generalizing—noticing patterns, articulating conjectures, representing how and why a general pattern holds—create a fertile context for the development of collective mathematical agency. It is content that has many entry points and can be accessed through multiple modes of participation. You can find a summary of the generalizations that students work on in the examples in this book in Appendix A. Teachers in the video were working from lesson sequences which we wrote in collaboration with another group of teachers. Interested readers can find the full lesson sequences in the book *But Why Does It Work?* (Russell et al., 2017).

Finally, we want to make clear that the mathematics content of these lessons is only one aspect of students' mathematical study—a deep dive into the structure of numbers and operations. It is not the only kind of mathematics investigation students should encounter. We advocate a rich mixture in students' mathematics curriculum that includes becoming fluent with a variety of calculation strategies, constructing and analyzing geometric objects, collecting and describing data, studying how people from different backgrounds and cultures have used mathematics, and undertaking projects to answer questions about the world using mathematics (National Council of Supervisors of Mathematics & TODOS, 2016). Important work is being done, including by contributors to this book, to create investigations in which students use mathematics to interrogate issues coming up in their own communities (e.g., Aguirre et al., 2019; Cirillo et al., 2016; see also Appendix B). All of these experiences help to raise students' voices with an emphasis on making sense, being curious, asking questions, and taking risks.

In this book, we are exploring the ways in which teachers support every voice in their classroom to engage with ideas about numbers and operations. These investigations are not limited to calculation but, rather, challenge and inspire students to delve into underlying mathematical structures.

How to Use This Book

Each chapter of this book focuses on an aspect of building a community that weaves deep mathematics with equitable participation and raises questions for reflecting on practice. The following suggestions will help users of this book to get the most from reading and viewing.

Do the Math

Because the mathematics of noticing, conjecturing, and representing general ideas about the operations may be unfamiliar, this resource offers readers an opportunity to investigate some mathematics for themselves as a way to introduce the concepts that students in the video are working on. We strongly suggest engaging with these investigations before watching the related videos in order to get a sense of the complexity of the ideas with which students are working and to better interpret what the students are saying and doing. (If you are like us, you may enjoy adding some math to your day!)

Watch the Classroom Video

One to three two- to eight-minute classroom video clips are the focus of each chapter. We can't say this strongly enough: *Watch the video*. It is possible to understand some of what is going on in the classroom by reading the text and transcript, but seeing the students—noticing their gestures and expressions, hearing the confidence or hesitation in their voices, waiting out the silences—are aspects that don't come through on the page.

When you watch a classroom video, it's tempting to quickly take on a stance of criticism, looking for what the teacher "should have done." Keep in mind that none of us viewing these short clips have the teacher's knowledge about their students or the context of the lesson—what happened before or what the teacher intends to do next. Viewing the video is an exercise in close observation: What do you notice about what students say and do? What does that imply about student learning? What do you notice about what the teacher says and does, and what do students say and do in response?

In most chapters, we recommend that you view each video clip twice, using different lenses. The first time through, focus on what students are learning and the way the teacher gives students access to deep mathematics. What are the ideas students are coming up with? How does the teacher respond to these ideas? In the second viewing, think about students' participation. What opportunities and supports enable different students to dig into the mathematics, express ideas and questions, and interact with other students and with the teacher in building ideas? Is there evidence that students are developing identity and agency as math learners? We provide specific Reflection Questions for these two viewings for each clip.

Transcripts for videos appear at the end of chapters for your reference as you reflect on and discuss with colleagues what you have seen. These are not intended to replace watching the video, since the transcript does not capture gestures, facial expressions, the length of pauses, or other factors that may be important in considering what is happening in the class. Further, while the audio of the video is generally good, and one of us was always making running notes of what students said while the other videotaped, it was not always possible to be certain of students' words. In these cases, we did our best to transcribe accurately, but there may be mistakes, and in a few cases, we have written "unintelligible" to indicate that we could not make out what a student said. There are three aspects of speech that you will hear in the video that we did not capture in the transcriptions: (1) some repetitions of words or phrases made by the teacher or student (e.g., both teachers and students often start a sentence, then restart it); (2) nonword interjections, such as "um"; and (3) variations of pronunciation (e.g., for "going to," people often say "gonna" or variations in between the two). The transcripts, then, are aids for remembering what you heard and saw in the video, but they are not substitutes for watching the video.

Read and Reflect on What Others See in the Video

We believe that we learn best in community—a community in which our tentative thoughts are welcome, in which we try to truly hear and understand others' ideas, in which we challenge each other and ourselves to think about our beliefs and actions, and in which we build stronger commitments than we might be able to sustain on our own. In that spirit, we invited the teachers in the videos and also a small number of educators, representing a variety of backgrounds and roles, to provide brief commentaries on the video clips. The six Collaborating Teachers and seven Critical Friends (introduced in the Preface) all bring to this work both a deep interest in mathematics teaching and learning and a focus on lifting up student voices, especially the voices of students from groups who have been historically marginalized. There is no simple list of strategies that, if implemented, will establish a dynamic mathematics community that includes every student. Rather, teachers' *ongoing, persistent, and determined reflection* on their classroom practice and its effects on student learning is the critical factor.

The purpose of the commentaries in each chapter is to open up different perspectives for viewing each video lesson. As the commentators viewed these classroom videos, they made different observations, raised different questions, and took away different ideas to apply to practice. Thinking about the reflection questions that follow each commentary can help you, alone or with colleagues, choose themes to focus on. Some of the commentaries will undoubtedly connect with issues you have already identified in your practice, while others may provide unexpected questions about aspects of teaching, learning, and participation that have been less visible to you.

In our own years of teaching and of collaborating with teachers over many decades, we, the authors, have consistently found that collaborating to reflect on practice is powerful. Teachers can and do raise questions about their own practice alone, but considering the observations and questions of other educators ensures that one's own questions are not restricted to established routines and beliefs. For that reason, we encourage you to find a partner or form a study group to reflect together on what can be learned from the students, teachers, and other educators who have contributed to this book.

Take Next Steps

Each chapter concludes with suggested "next steps" for you to try in your own practice. We have written these suggestions with the hope that you will explore new ways to ensure equitable participation of your students while maintaining your commitment to deep mathematics.

Notes Organizer

The Notes Organizer is an electronic supplement found at companion.corwin.com/courses/equitable-deep-math. It provides the math activity, Reflection Questions, and Next Steps from each chapter, with space to take your own notes.

Facilitator's Guide

The Facilitator's Guide (also available at companion.corwin.com/courses/equitable-deep-math) is designed for those who lead professional development or a study group based on this book. The guide suggests a chapter-by-chapter plan for organizing study group meetings and offers tips for facilitators.

A Note About Some Terms We Use

There are some terms common in the education community that can refer to different things. To avoid confusion, we specify here what we mean when we use the terms *mathematical practices* and *mathematical representations*.

By *mathematical practices*, we mean the working practices of professional mathematicians. A number of different documents, such as the National Council of Teachers of Mathematics *Principles and Standards* (NCTM, 2000) and the *Common Core Standards* (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), have offered lists of practices. Our use of the term includes these but is not limited to them. For example, two key mathematical practices illustrated in this book are *noticing patterns* and *formulating conjectures.*

Mathematical representations are physical, visual, or verbal depictions that embody a mathematical object. Mathematical representations may include pictures, diagrams, number lines, graphs, arrangements of physical objects, mathematical expressions, equations, or the statement of a generalization. Because story contexts can carry so much meaning about mathematical relationships at the elementary level, we include them as representations as well. *Student-created mathematical representations* are drawings, diagrams, models, story contexts, and so on, that come from students' imaginations, as well as more standard forms of representation, such as number lines or arrays, that students have incorporated into their repertoire.

Reflection Question

This resource is structured around four aspects of mathematics community: *every voice matters; collaboration supports student agency; student-created representations offer anchors, openings, and depth;* and *students are initiators and advocates for their own learning.* Think of a recent experience in your own context. How were each of these aspects present? If you were to choose one on which to focus right now in your own work, which would you choose and why?