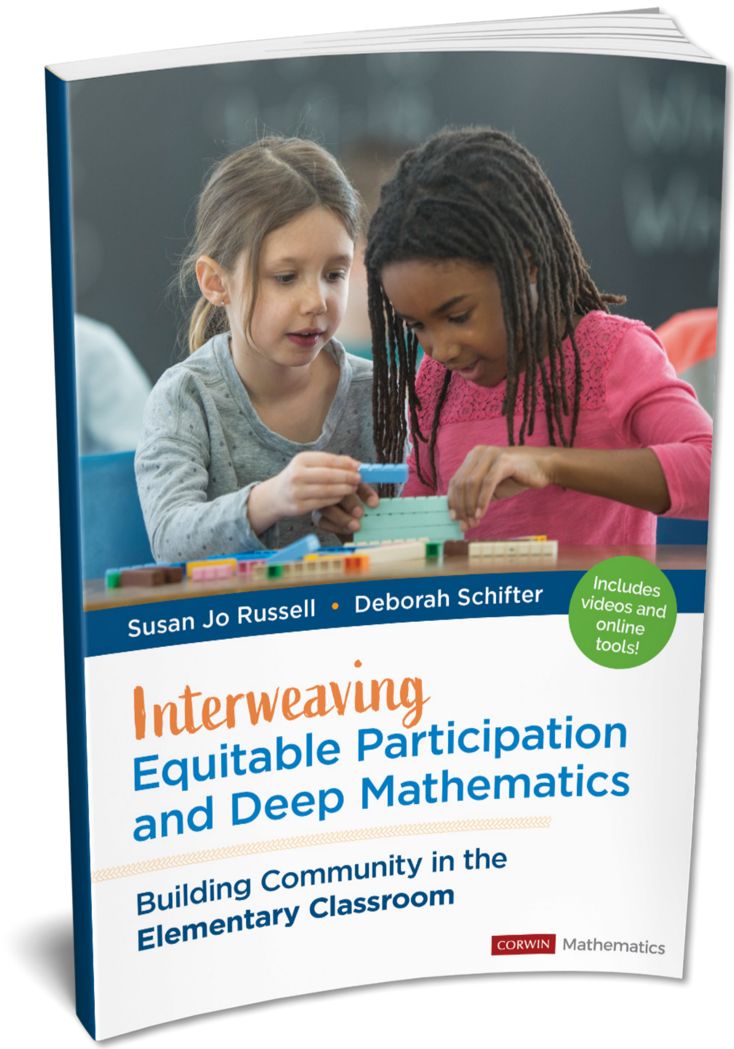

Thank you

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Please enjoy this complimentary excerpt from Interweaving Equitable Participation and Deep Mathematics.

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CORWIN

Chapter 1

Creating Multiple Openings Into Engaging Mathematics

Our work is based on two assumptions:

1. Mathematics is an interwoven network of ideas.
2. Students come to school with mathematical ideas, and part of the work of the teacher is to draw out those ideas and help students develop them further.

Students in the elementary grades are learning not only mathematics content but also what mathematics is and what it means to be mathematical thinkers. We view the learning of mathematics as an active endeavor—the construction of ideas, rather than passive absorption of delivered knowledge. As students find their place in the mathematics classroom, we want them to learn that they are capable of making sense of complex mathematical ideas. We want mathematics classrooms to be settings where students have the opportunity to develop positive and productive mathematical identities as doers of mathematics. We hope for mathematics classrooms that are contexts for developing mathematical agency, where students learn to investigate mathematics, share their own ideas, interact with classmates' ideas, and take responsibility for their own learning.

Given this description of what a mathematics classroom can be, we pose, again, the question we asked in the Introduction: How do we ensure that every student contributes a voice to a mathematics community, including students who have been historically marginalized in mathematics, students who have not believed they have mathematical ideas that are important to share, or who, when they have tried to express their ideas, have not been heard?

In this chapter, you will

- do some math for yourself to become familiar with the ideas you'll see students working with,
- watch a video to explore how first-grade students are discovering their own mathematical agency and identities in a whole-class discussion,
- read what our Critical Friends and the classroom teacher have to say about students' learning and participation, and
- consider how to create openings to ensure that every student contributes their voice to the mathematics community.

Whole-class discussion is one of the primary forums in which students are invited to use their voices to contribute mathematical ideas. Take a moment to reflect on your own experiences as a participant in a discussion. Can you remember a time when you have been in a group that seemed closed to your ideas? Did a few people dominate the discussion? Did others seem more certain, more on top of what they had to say? Did you worry that you weren't smart enough or experienced enough to contribute? Or were you usually eager to get out your own ideas? Did you try to understand others' ideas? Was there enough time to hear multiple perspectives?

Now think about what a classroom mathematics discussion might feel like from the perspective of an elementary-school student. Is it a setting in which students are willing to put out tentative ideas, clarify them, ask questions, and build mathematics together? Might it be intimidating for some students? What does it take to create a classroom community that works on deep mathematics and also supports students in participating with all their different personalities, backgrounds, facility with language, and mathematical understanding?

As you consider the classroom video and the commentaries in this chapter, we'd like you to think about three aspects of the principle that *every voice matters*. These three aspects of building community lay a strong foundation for interweaving rigorous mathematics and equitable participation.

1. The mathematics content is powerful, engaging, and challenging, but there are many entry points into the ideas.
2. Accessing the depth of the ideas takes time. “Productive lingering” on a few related questions allows students to dig deeply into mathematics concepts.
3. Teachers can create multiple openings into the mathematics. Students can have voice in different ways.

Do the Math

As we explained in the introduction, you’ll get more out of the children’s thinking in the video if you do some related math work before viewing. Even though you may find the mathematics itself familiar, doing some work of your own with the same mathematical ideas with which the students are working will help you understand its complexity and importance.

1. Make a drawing or diagram or build a physical model for each of these equations:

$$3 + 6 = 9$$

$$9 - 3 = 6$$

$$9 - 6 = 3$$

   Can you draw or build a single representation that shows all three of these equations?

   Explain to a colleague how your representation shows all three equations.
2. The three equations are an example of a big idea about the relationship between addition and subtraction. Can you write an “if . . . , then . . . ” sentence that expresses the general relationship among these three equations?

   If . . . , then . . . .

   Can you use your representation (drawing, diagram, or physical model) from Question #1 to explain why your “if . . . , then . . . ” sentence is true?
3. Share your “if . . . , then . . .” sentence with a colleague. What is the same about your sentences, and what is different? How do your representations demonstrate why your statements are true?

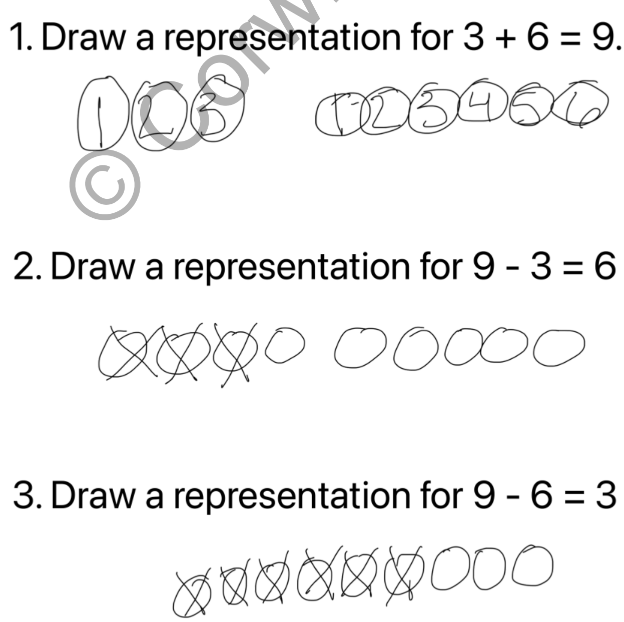
Watch the Video: “Where Do You See the 3?”

To begin to understand what it looks like to create openings for every student, let’s watch and analyze Video 1.1. This lesson is from a sequence of lessons in which Natasha Gordon’s first graders are investigating the relationship between addition and subtraction, a complicated idea for young children who are new to thinking about these operations. In order to understand the idea, not only must students conceptualize what each operation does, but they must hold in mind images for *both* operations *at the same time* to understand how they are related. At first, students notice this relationship with specific numbers, but then they gradually consider how it applies to all the numbers with which they are working.

In the previous lesson, Ms. Gordon asked students to create a representation for each of the three equations: $3 + 6 = 9$, $9 - 3 = 6$, and $9 - 6 = 3$. The activity sheet also included the question, “What is the same in your representations, and what is different?”

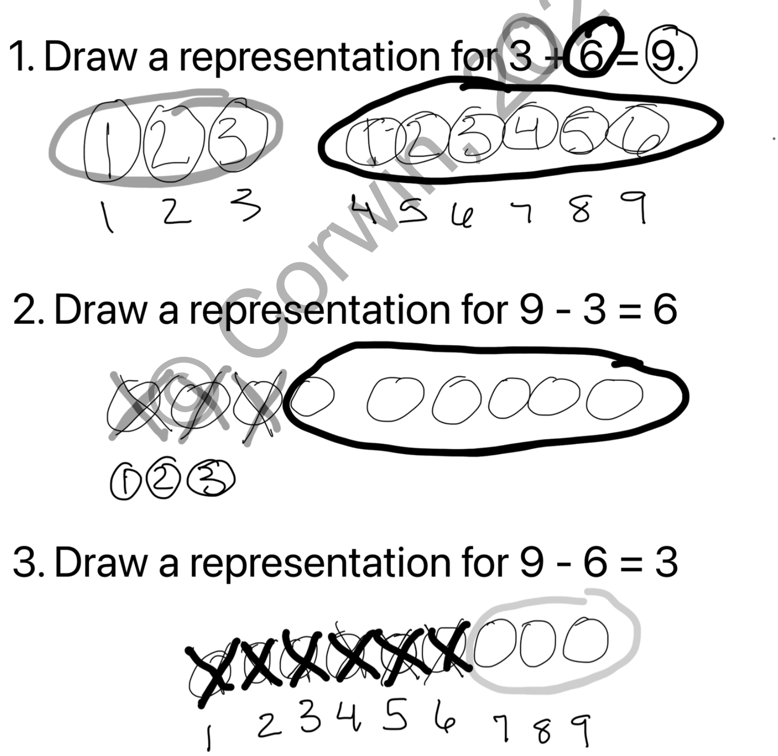
For the lesson shown in the video, Ms. Gordon selected a piece of student work (Figure 1.1) to project onto the whiteboard and discuss with the class.

Figure 1.1 • A First Grader’s Work Representing $3 + 6 = 9$, $9 - 3 = 6$, and $9 - 6 = 3$



Before the part of the lesson you're going to view, the class talked about what they noticed is the *same* in the three equations and the three representations. The whole-group discussion was paused for a few minutes while students talked with partners about what they noticed—a classroom structure commonly referred to as turn-and-talk—and then students shared their ideas in the whole group. As they spoke about what they saw as similar in the three representations, Ms. Gordon noted their ideas by using colored markers on the whiteboard. She used one color to indicate groups of 3 in each of the representations (shown with gray in Figure 1.2) and a different color to indicate the groups of 6 (shown in black). By the end of the day's discussion, of which this video clip is a part, the whiteboard looked like Figure 1.2.

Figure 1.2 • Ms. Gordon's record of what students noticed about what is the same in the three equations and representations. She used different colored markers for the groups of 3 and groups of 6, shown here as gray and black, respectively.



During the discussion about similarities, one student, Livia, pointed out that the numbers are in a different order in the three equations. We'll join the group as Ms. Gordon comes back to Livia to ask her to elaborate her idea.

You'll view the 5-minute video clip twice, with different lenses, in order to help you think about weaving together deep mathematics content and openings for students' voices.

First Viewing of the Video: The Mathematics Students Are Working On

Video 1.1

"Where Do You See the 3?"

qrs.ly/fqfs4v8

To read a QR code, you must have a smartphone or tablet with a camera. We recommend that you download a QR code reader app that is made specifically for your phone or tablet brand.

Watch the video clip, "Where Do You See the 3?," with a focus on the mathematics content students are learning.

Reflecting on the Video: The Mathematics Students Are Working On

[You may want to use the transcript at the end of this chapter as you consider these questions.]

1. What are the important mathematical ideas in this clip?
2. How are students engaging with these ideas? What different ideas are students working on? Are there different entry points into the mathematics?
3. In what ways is the mathematics challenging and engaging for the students?
4. What does the teacher do to focus the discussion and to promote persistence with complex ideas?

Second Viewing of the Video: Finding a Way in Through Multiple Modes of Participation

Watch the video clip, "Where Do You See the 3?," with a focus on how students have voice during this discussion.

Reflecting on the Video: Finding a Way in Through Multiple Modes of Participation

[You may want to use the transcript at the end of this chapter as you consider these questions.]

1. What do you notice about different modes of student participation? Are there opportunities for students to find different openings into the discourse?
2. How does the teacher support students' voices in this discussion?
3. How does the posted student work and Ms. Gordon's use of colored markers support students to find ways in to participate in the whole- group discourse?
4. If you were the teacher in this classroom reflecting on this lesson, what might you want to make note of in order to strengthen student participation? Are there aspects of the lesson that worked well to create openings into the mathematics? Are there questions you have about how you could better encourage students' voices and help students develop agency as mathematicians?

Read and Reflect on What Others See in the Video

Let's return to the three aspects of the principle that *every voice matters* listed at the beginning of this chapter:

1. The mathematics content is powerful, engaging, and challenging, but there are many entry points into the ideas.
2. Accessing the depth of the ideas takes time. "Productive lingering" on a few related questions allows students to dig deeply into mathematics concepts.
3. Teachers can create multiple openings into the mathematics. Students can have voice in different ways.

In this section, you will encounter reactions from some of our Critical Friends to the class session—what they notice and questions they raise. Also included is commentary from Ms. Gordon about how she was thinking about students' entry into the mathematics content during a similar lesson.

1. Critical Friends Consider How These Young Students Encounter the Mathematics Content

"Fact families" are a familiar topic in elementary classrooms. If you search for "fact family" online, you find many worksheets in which children are asked to fill in a series of blank equations with sets of numbers such as 3, 6, and 9. But, without a focus on making sense of these relationships, students might learn to fill in the blanks correctly without thinking about how and why these equations are related.

In Ms. Gordon's lesson, students explore the relationship between addition and subtraction by digging deeply into one example. Young students first need to understand each problem in itself. Many students in first grade are still solidifying the idea that addition can be seen as joining and subtraction as removing, and they are working through what each component of an addition or subtraction equation stands for. Students are also learning what it means to represent a mathematical equation: How do the circles and x's in the drawing represent adding or subtracting? How are the drawings and the equations related to each other? By tracking how each of the three quantities appears in the equations and in the drawings—different ways of seeing how 9 is composed of 3 and 6—students are also taking a step toward understanding more generally how addition and subtraction are related. Because of the many different components of this complex idea, different students might be working on different aspects of the concept within the same lesson.

Here is what some of our Critical Friends have to say about the math in this lesson.

Hetal Patel: The teacher sticks to the math and allows the students to describe it. There's a lot of descriptive language that the kids are using, and she sticks to the words they use. The students have multiple opportunities and a variety of ways to use that descriptive language for what they are trying to make sense of for themselves or for each other.

Virginia Bastable: It feels important to say, Don't wait until you think the idea can be solid for the students. You can help them begin to think about a messy idea. None of the kids seem upset that they're not saying the whole idea. There's nothing negative going on. They're sort of playing with this idea. And even if you're not sure they're going to consolidate it in whatever time period you have, it's still worthwhile. I think that's an important message for people, especially in this day and age when every 20 minutes you have to check off that you met some objective!

Darlene Ratliff: For young students, the mathematics can be murky. Here's how a first-grade class "murks" through it. Nevertheless, the ideas still emerge during the lesson.

Reflecting on the Mathematics

1. Hetal Patel notices how the teacher makes use of students' own words. Refer to the video or the transcript. How does the teacher encourage and accept students' own language? How does she help students expand and clarify their language, both for themselves and for the understanding of other students?
2. What is your response to our Critical Friends' thoughts about engaging young students with challenging mathematics that is "messy" or "murky" for them or that you may not be able to bring to closure? What do these observations have to do with students developing their identities as doers of mathematics?

2. The Teacher Reflects on a Similar Lesson: What Are Students Learning?

The following year, Ms. Gordon taught the same lessons in her first-grade class. Students represented and discussed the three equations involving 3, 6, and 9 in a lesson similar to the one you watched on video. Based on what she observed in that lesson, she planned the next lesson to focus on two related story problems, using the numbers 6, 9, and 15. This is an example of “productive lingering”—investigating a small set of related problems thoroughly in order to dig into significant mathematics, in this case, the relationship between addition and subtraction. Rather than practicing with pages of sets of problems similar to $3 + 6 = 9$, $9 - 6 = 3$, and $9 - 3 = 6$, students have the opportunity to examine another single example—allowing for time to draw, discuss, and ask questions. The ideas are developed across several lessons. Afterward, Ms. Gordon reflected on the students’ representations and discussion of the story problems.

Natasha Gordon: In the session about 3, 6, and 9, students stated that the subtraction equation MUST have a specific answer because of the similarities in numbers in the addition equation. I wondered whether they understood what was happening with the numbers as they related to the actions of addition and subtraction. Going into the next session, I was curious as to whether students would be able to better articulate their understandings and connections, as well as if any new ones would emerge.

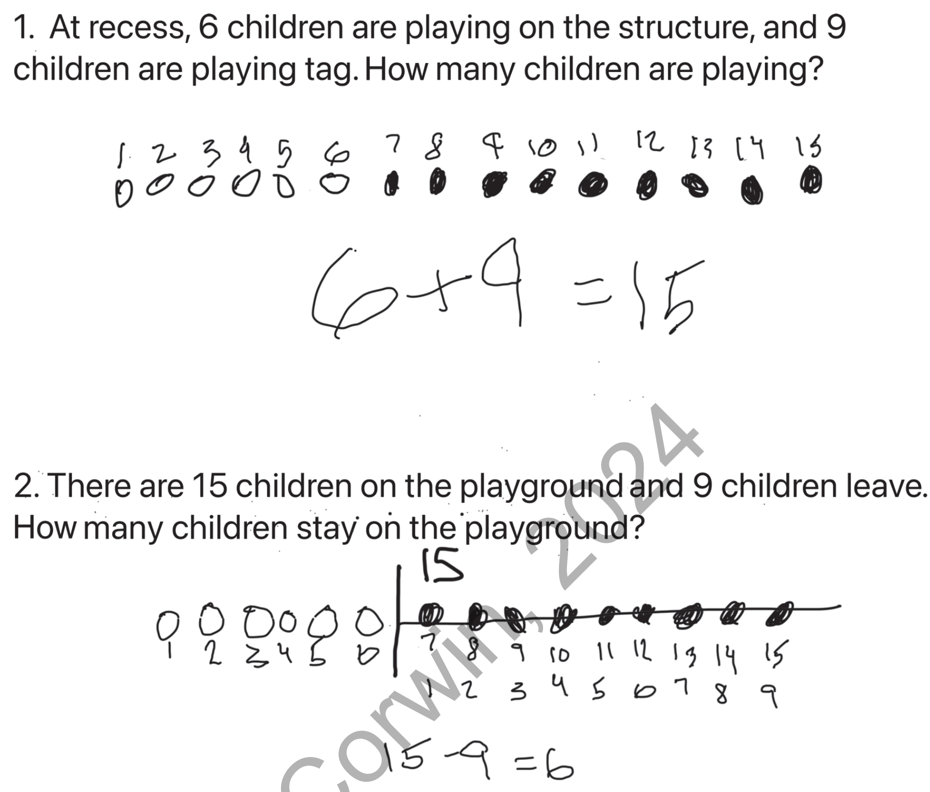
For this session, students drew representations for the following problems:

1. *At recess, 6 children are playing on the structure, and 9 children are playing tag. How many children are playing?*
2. *There are 15 children on the playground, 9 children leave. How many children stay on the playground?*

Students were asked whether the first problem helped them solve the second problem and, if so, how?

When we came together to analyze one student’s work (Figure 1.3), the discussion was filled with many student voices as we tried to make sense of each problem and each representation. When I asked whether the first problem could help you solve the second problem, I heard the following comments.

Figure 1.3 • A Student's Work Illustrating Two Related Story Problems



Reynald: Yes, because 6 + 9 = 15 and 15 – 6 = 9, because if you have 15 and you take away 9, it will equal 6 because 6 + 9 = 15.

Ms. Gordon: Can someone add onto Reynald? How is that helpful if you know 6 + 9 = 15? He said then 15 – 9 will equal 6. How is that helpful?

Xavier: Both problems, both have 15, but on the bottom one of the problems, the other 9 was taken away, and the first one they were still there, so they went somewhere else.

Ms. Gordon: Who can say more? Xavier pointed out that there were 15 children in both problems. We can see the 15 circles in both problems, and also I see 15 in both equations. But you said in one problem the 9 is taken away? And then in the other problem what happened with the 9?

Xavier: The 9 are still playing.

Ms. Gordon: OK. So in one problem the 9 are taken away, and in the other problem the 9 are still there. Someone else say more . . .

Tatianna: I was talking about this with my talk partner because it *does* make sense because this one is starting with 6 + 9 and it equals 15, and if it's 15 and it takes away 9, it will leave back where we started with 6.

Ms. Gordon: Interesting. Something about this problem [points at top problem and underlines the 6 in the equation $6 + 9 = 15$] we *started* with 6. And you said this problem [points at bottom problem and underlines the 6 in the equation $15 - 9 = 6$] we're *left* with 6. So how is that helpful? Someone, add on even more. Thank you, Tatianna. This problem we started with 6, and we ended with 15 children altogether. And with this problem, we started with 15 children, and we're left back with 6, Tatianna said. Miriam, do you want to add on?

Miriam: I just want to say it's like you're putting the stories together because if 15 children were playing on the playground, and 6 were playing on the structure and 9 were playing tag, you can put both stories together by, after a little bit of them playing, 9 children left.

Ms. Gordon: But can you say more? How can you put those two together? So you said after some time . . .

Miriam: It's like, it's basically like 9 children come in on the playground and then those 9 leave.

In this five-minute dialog, four students built on each other's ideas and collectively found a connection between the two problems: that the nine students who came onto the playground to play tag could be the same nine students that left the playground, leaving us with the original six children. Tatianna stated that "it does make sense" as she added on to her peers' ideas and tried to explain the connection further. Her exclamation was a sign of her bringing the math together and tying it into the context of the problem. I do not believe all students have arrived at this understanding, but I do believe that as this relationship is explored further, more students will approach this understanding.

Reflecting on Ms. Gordon's Writing

1. As Renald, Xavier, Tatianna, and Miriam discuss the problems about children on the playground, how do their comments build on one another? What are they saying about the relationship between the two problems?
2. What is your response to Ms. Gordon's last sentence in her comments, "I do not believe all students have arrived at this understanding, but I do believe that as this relationship is explored further, more students will approach this understanding"?

3. Critical Friends Consider How Students Are Offered Multiple Modes of Participation

A teacher's responsibility to promote equitable participation is complicated. When using video clips in this book to reflect on our own practice, we are seeing a very small slice of classroom activity. We do not know what the teacher knows about what came before what we see, how the teacher is supporting the needs and strengths of individual students, or how the teacher plans to follow up. Some aspects of what happens on the video are invisible to us. For example, when Ms. Gordon later reflected on the lesson, she commented, "When we broke into turn-and-talk, I made a point of visiting pairs of students who had not yet spoken up. I noticed that four of those students subsequently contributed to the whole-group discussion."

We use these brief video clips not to comment on what the teacher "should have done," but as learning tools for ourselves—to reflect on what we see, to notice which teacher moves bring students into the conversation, to ask questions and wonder what we might do to ensure equitable participation in our own contexts. Even without knowing all that the teacher knows, we can use these short excerpts to provoke questions for our own practice, such as these that our Critical Friends and Ms. Gordon's second-grade colleague, Quayisha Clarke, raise. Note that they viewed the full 30-minute lesson from which this clip is taken.

Quayisha Clarke: Within the turn-and-talk and within the discussion, Natasha [Gordon] would ask questions like, "So what do you think about Livia's idea?" It's really evident to the students that their ideas matter because the teacher is listening to them, and so are the students. What everyone says matters because they're going to talk about this new thing that just happened. I think that gives a lot of power to student voices. I counted, too, and I think it was almost everyone except two students who talked in the whole-group discussion. That doesn't just happen. I'm not going to just walk into any classroom and see all of the students talk in a 30-minute math discussion. She's given access to everyone's voice, and power, as well.

Virginia Bastable: When Ms. Gordon does call on people, she gives the students a lot of time to talk. Even if they're stumbling or not saying their ideas well, there's clearly time for them to get through that, and sometimes they do and fix it and sometimes they say, come back to me, I can't finish. I thought that was supportive of increasing voice.

Darlene Ratliff: As one often sees in first-grade classrooms, boys fidget and move all over the place in this lesson. But they're still engaged. I want to emphasize that they're still engaged, even though they are moving around and fidgety. The teacher didn't spend all of her time saying, "Come on, sit down, stop rocking, stop playing." It was all about the mathematics and the discussion, and not about who's fidgeting. Whether they're fidgeting or not, if they're still tuning in to what you need to have happen, that's an important place for those students.

I also want to say that sometimes voice isn't talking. Sometimes, voice is signaling. As we watched this class, I can pretty much say all of them displayed voice in some way, whether in the turn-and-talk or gestures, everyone had some voice. Some had more voice than others, but overall, there was an atmosphere of voice.

Lynne Godfrey: I was looking at the different kinds of questions the teacher posed. It's one thing to go up to the board to point. But I'm wondering, who gets to answer the thinking questions?

Cindy Ballenger: Josiah was so clear and articulate. It was my feeling that he was helping everybody. So then there's the question, Does the teacher always use Josiah in this way, or are there other kids who take on this role? There might be moments when Josiah is the guy the teacher needs to go to, but you have to take a long view.

Yi Law Chan: I noticed a core of eager hands, the very eager waving to signal something they very much wanted to share. I'm also thinking about how the role of those kinds of signals plays out in a classroom that also has other kinds of signals, as well as opportunities for students to do the turn-and-talks. How will these different nonverbal means of communicating impact different students? What I noticed is that there are definitely students along the periphery, physically and also in their involvement in verbalizing, in the whole group. The turn-and-talk allowed every pair of students to verbalize thinking, but I did notice that there's a central group of students who had the floor more often in the whole-group conversation. So I have curiosity around the voice there in the whole-group setting. How do we assess the impact of a student's contribution on their own learning or the impact of others' comments on their learning? It's a question I have that I'd like teachers to think about. When I visit a classroom, I might not be able to assess this impact, but teachers in the classroom have the ability to assess that over time.

Virginia Bastable: Paying attention over time is such an important theme for a teacher. In any one clip, we're seeing so little that it's hard to make these judgments. But as a teacher in a classroom, I need to notice what kinds of questions I am asking and to whom and make sure those thinking questions get more distributed.

Reflection Questions

1. What do you notice in the clip from Ms. Gordon's class about how students have voice? Are different modes of participation evident? Is it important for every student to speak in whole group? How else might students productively

(Continued)

(Continued)

participate in the whole-class discussion? What do you think, in your own context, about Darlene Ratliff's statement that "Sometimes voice isn't talking"?

2. While many students have the opportunity to participate in different ways, in this whole-group discussion, there may still be students who do not find an opening to participate, students who, as Yi Law Chan mentions, are on the "periphery." You may have noticed students in the video—and it is likely this would be true in any classroom—who don't seem to actively participate. What would you want to know about these students? How might you follow up with them? What might you plan in order to provide openings for them to engage in the next whole-group discussion?

What Do You Want to Remember From This Chapter?

Take a few minutes to note for yourself ideas you want to hold onto as you continue to investigate the meaning of a mathematics community and how to build it. What teacher moves have you noticed in this chapter that you want to bring into your own practice? Here are some of the ways we like to think about what the teachers and Critical Friends in this chapter have said about creating openings for every student:

- **Create multiple entry points and openings.** A large part of equitable mathematics teaching is access. Give students openings to enter into mathematical ideas through a variety of representations, including words, equations, and diagrams, to ensure the greatest possible participation.
- **Create an expectation of "productive lingering" on important ideas.** Structure lessons to focus on one or a few related questions. Encourage students to investigate those questions deeply, welcoming questions and ideas from many voices.
- **Celebrate different forms of participation in class discussion.** Participation in math discussion can include stating what you notice, asking questions, gesturing, building on classmates' ideas, and indicating agreement, disagreement, or confusion. Make openings for, acknowledge, and celebrate all of these forms.
- **Pay attention over time.** Develop ways to track which students are participating and in what ways. Who speaks up in whole-group discussions? What kinds of questions are you asking to which students? Who is sharing their work? Who is commenting on other students' ideas or representations?

Taking a Next Step

List the students in your class and place their names in one of three groups: (1) one-third of the students who participate most in math class, (2) one-third of the students who participate least in math class, and (3) those who fall between the two. As you look over the names in the three groups, what do you notice? What questions do your lists raise for you? If you don't have your own classroom, adapt this activity to another group with which you are working.

Video 1.1 Transcript: **"Where Do You See the 3?"**

Ms. Gordon: Livia, I'm going to come back to you because you started talking a little bit about how they're different. Can you say more about how they're different? What were you saying?

Livia: So what I was saying was that they're the same numbers but they're switched up, like in the first equation it says 3 plus 6 equals 9. And then the next equation is 9 minus 3 equals 6. And then the next is 9 minus 6 equals 3.

Ms. Gordon: How many friends also see that, that they're the same numbers but they're a little bit different because they're in different orders? For example, Caitlyn started talking about the number 3. Where do you see, in the first representation, where do you see 3? Audrey?

Audrey: The first 3 in the first one.

Ms. Gordon: The first three circles. But in the second representation, where do you see the 3? J'aimeson?

J'aimeson: [Points to board.]

Ms. Gordon: So, these three right here. Audrey, can you say a little bit more? What did you just say?

Audrey: I said the ones that were taken away.

Ms. Gordon: So how is that different? Just in those first two. For the first problem we noticed that the 3 is represented by the first three circles. But in the second problem the 3 is represented by the three that are being taken away. Josiah?

Josiah: Because that one is adding 3, and that one is taking away 3.

Ms. Gordon: How can you tell that this one is taking away 3?

Tierra: Because it says it's taking away 3.

Ms. Gordon: Where?

Tierra: Right here.

Ms. Gordon: Ah, in the equation. 9 take away, 9 minus 3. How is that represented in the work? What did this scholar do to represent 9 take away 3 or 9 minus 3?

Anu: They x’d it.

Ms. Gordon: Ah, they crossed them out. Okay, so we talked about 3, we noticed 3 is represented by the first three circles in the first equation; 3 in the second equation is represented by the three that are taken away. What about the third representation? We said we saw 3 there, 3 is in this equation and in the representation, but where is the 3 in this representation? Tunmiche? What does the 3 represent in the third equation?

Tunmiche: Right there.

Ms. Gordon: Where?

Tunmiche: [points to board]

Ms. Gordon: Can someone say more? Okay, so we see the three are right here. Can someone say more? Josiah, I see that you want to add on.

Josiah: The 3 is represating for what it equals.

Ms. Gordon: OK, what it equals or . . . ?

Anu: The answer.

Ms. Gordon: Who can say more—what it equals, the answer . . . Audrey?

Audrey: Like what it means.

Ms. Gordon: Josiah?

Josiah: The total.

Ms. Gordon: Okay? Interesting, you’re saying the total. When we say total, the total is how much we have . . .

Students: Altogether.

Ms. Gordon: Is the 3 what we have altogether?

Students: No

Ms. Gordon: So what is this 3? Janiya?

Janiya: What we take away?

Ms. Gordon: Okay, so we know this problem is subtraction. We are taking away, but what does this 3 represent?

Anu: What remains?

Ms. Gordon: So, in the first problem the 3 is what we started with. In the second problem we took away 3. And in the last problem 3 is what we’re left with.