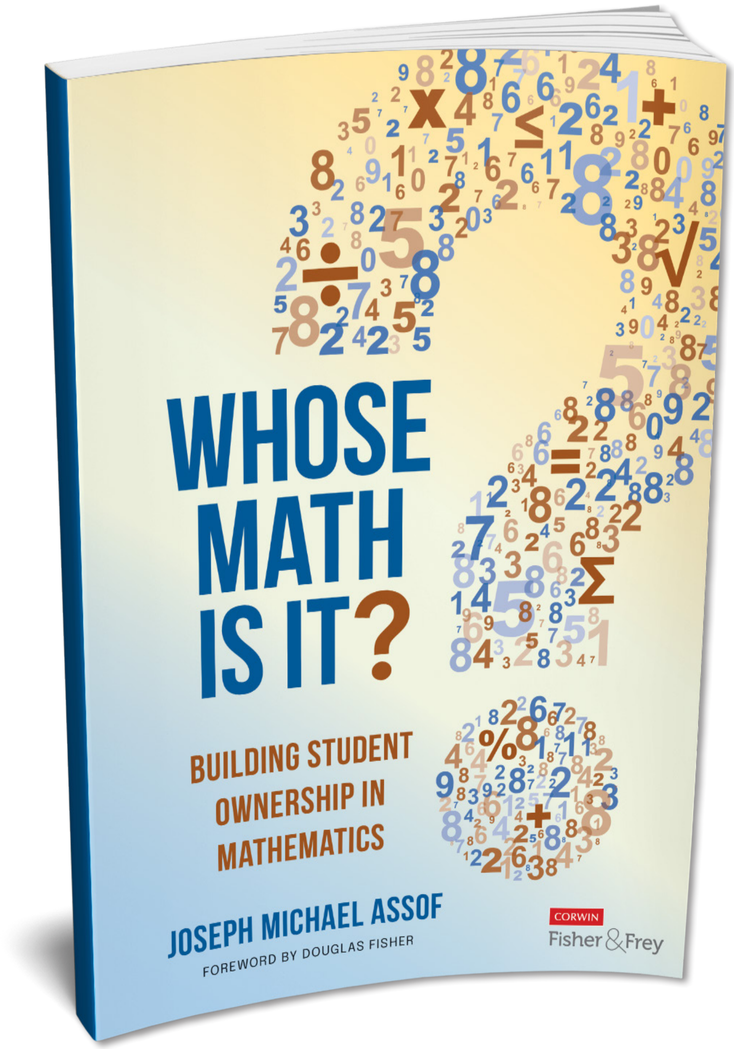

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Introduction

Keeping the End in Mind

Imagine a mathematics classroom where students are not only actively engaged in critical thinking, problem solving, and constructive argumentation, but where they are also aware of their own learning, seek feedback on their work, provide feedback to their peers, and monitor their own progress. One can imagine further that there exists a culture in this classroom of high expectations sustainable only by its equally prominent culture of support.

This is a classroom fueled by efficacy—where students are choosers and users of learning strategies that have proven effective for them in the past and thus give them confidence to use them again. This is a classroom where the teacher may truly embody the role of facilitating learning, with confidence that their expertise is not going underutilized. Now, compare this abstract ideal to the concrete reality. This comparison might tempt some down the student-by-student road; checking off individual talented students who could rise to the occasion of such an idealized classroom and crossing off others who likely would not. This approach, however, begs the question: Do we develop or select talent? And while many of us in education might instinctually and fervently (and commendably) react to such a question, without efficacy of our own, the prospect of developing such a high degree of talent might seem unattainable.

Thus is the purpose of this text. This book seeks to act as the representational intermediary between the abstract ideal classroom described above and the concrete realities of our own classrooms. This text is designed to help mathematics teachers realize the ideal and bring the abstract to the concrete through key practices targeting the development of student ownership of learning. For when asked the question *Whose math is it?* every student should respond, *My math!*

The Role of the Students

Think about the students in your classroom. How do they see themselves as participants in the mathematics classroom community? Further, how do they see themselves in respect to math itself? Some students consider themselves to be passive recipients in the mathematics classroom—why is this? Math, to them, is likely a large collection of facts and procedures that need to be unveiled by an expert so they can be apprenticed into recall and reproduction. In this sense, mathematics is much like tradition in that it must be passed on to survive—if all the math teachers suddenly vanished we would never know math again! (Something that would likely land with minimal tragic impact to the students described here.) These students don't have a *say* in mathematics—no one does! Mathematics just simply *is*.

Contrast the mindsets of these students with those in the classroom previously described, where students are clearly positioned as problem-solvers with agency over their learning. They have a stake in the game, they lean into challenge, and they believe progress will come with effort. To those with agency in the subject, mathematics is something that can be—and *needs to be*—discovered individually and collectively. The ability and authority to validate mathematical claims, check the accuracy of calculations, and determine the reasonableness of solutions lives within them—not beyond them. They may appreciate external validation, but it is not prerequisite to confident progress. To these individuals, math is *personal*, math is *owned*. These individuals cannot be told that $1 + 1 = 47$, for they have independent access to the existential structure of mathematics where this falsehood doesn't pass the smell test. Simply put, these individuals are mathematicians.

Surely we have had students arrive in our classrooms with mindsets on both ends of the spectrum outlined here—as well as in many places in between. The question for us as teachers becomes, how do we take students from wherever they are and help them develop more of the ownership required to be successful in mathematics? In order to do this, however, we need a benchmark understanding of their foundational starting point. One way to do this is by using the *Student Mathematical Ownership Itinerary* (Table I.1 and Table I.2). This tool can be used to inform you (and your students) how each learner situates themselves in the mathematics classroom and in respect to math itself. It can be used at the beginning of the school year as a pre-assessment of mathematical agency, as a formative benchmark throughout the school year to inform your instructional decision making, and at the end of the year to measure the impact of your approach.

Table I.1 Student Mathematical Ownership Itinerary

STUDENT MATHEMATICAL OWNERSHIP ITINERARY			
State the degree to which you agree with each statement below.			
1. I can use math as a tool to make sense of the world.			
[ ] Strongly Agree	[ ] Agree	[ ] Disagree	[ ] Strongly Disagree
2. Math is a large collection of facts and procedures that need to be memorized.			
[ ] Strongly Agree	[ ] Agree	[ ] Disagree	[ ] Strongly Disagree
3. I can discover math on my own.			
[ ] Strongly Agree	[ ] Agree	[ ] Disagree	[ ] Strongly Disagree
4. I need a teacher to show me how to do math before I can learn it.			
[ ] Strongly Agree	[ ] Agree	[ ] Disagree	[ ] Strongly Disagree
5. I can make choices when doing math about how I want to solve a problem.			
[ ] Strongly Agree	[ ] Agree	[ ] Disagree	[ ] Strongly Disagree
6. There is one right way to do math.			
[ ] Strongly Agree	[ ] Agree	[ ] Disagree	[ ] Strongly Disagree
7. I can check my own work to see if I did it right.			
[ ] Strongly Agree	[ ] Agree	[ ] Disagree	[ ] Strongly Disagree
8. I need a teacher to tell me if my answers are right.			
[ ] Strongly Agree	[ ] Agree	[ ] Disagree	[ ] Strongly Disagree

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Table I.2 Student Mathematical Ownership Itinerary (version 2)

STUDENT MATHEMATICAL OWNERSHIP ITINERARY			
Read each statement. Circle the picture that matches how you feel.			
1. I can use math as a tool to make sense of things around me.			
Strongly Agree	Agree	Disagree	Strongly Disagree
2. Math is a group of facts and steps to take that I need to memorize.			
Strongly Agree	Agree	Disagree	Strongly Disagree
3. I can figure out math on my own.			
Strongly Agree	Agree	Disagree	Strongly Disagree
4. I need a teacher to show me how to do math before I can learn it.			
Strongly Agree	Agree	Disagree	Strongly Disagree
5. I can make choices when doing math about how I want to solve a problem.			
Strongly Agree	Agree	Disagree	Strongly Disagree
6. There is only one right way to do math.			
Strongly Agree	Agree	Disagree	Strongly Disagree

STUDENT MATHEMATICAL OWNERSHIP ITINERARY			
7. I can check my own work to see if I did it right.			
Strongly Agree	Agree	Disagree	Strongly Disagree
8. I need a teacher to tell me if my answers are right.			
Strongly Agree	Agree	Disagree	Strongly Disagree

Source: Smiley icons courtesy of iStock.com/Makrushka

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To score this assessment, assign a scoring scale of 3: Strongly Agree, 2: Agree, 1: Disagree, and 0: Strongly Disagree to all odd numbered statements and a reversed scale of 0: Strongly Agree, 1: Agree, 2: Disagree, and 3: Strongly Disagree to all even numbered statements. Scores of 0–10 indicate low perceived ownership of mathematics, 11–16 indicate a moderate ownership of mathematics, 17–24 indicates a high level of student ownership of mathematics.

To be clear, I am not trying to send the message that students arrive in some sort of a fixed manner regarding mathematical ownership whereby some have it and some simply do not. Rather, this initial focus on the role of the student is meant to highlight the impact of their surroundings and learning environments—including their teacher—on their presumed capacity for mathematical ownership. In other words, as teachers, we have great influence over how students position themselves with mathematics. The language we use, the environments we foster, the tasks we launch, the ways we interact with others—all of this impacts how students are positioned in the content and our classroom/math course. That's great news! It means we have the power to affect positive change in our students' sense of self. If, that is, we act with intention. In the next section, I will seek to further illustrate how our decisions and actions as teachers produce much more than just marks on papers.

The Role of the Teacher

Think about our primary role as teachers of mathematics. Are we disciples of the subject, facilitators of learning, or perhaps, both? Consider the following exchange between a student and teacher during a middle school lesson on using variables to represent quantities in a real-world problem. Students are independently working on the following problem while the teacher circulates the room.

The perimeter of a rectangular swimming pool is 54 meters. The length of the pool is 6 meters. What is its width?

Student: *[raises their hand and signals the teacher over] Is this right?*

Teacher: *Can you tell me what you did?*

Student: *OK. Well, I wrote* $6 \cdot w = 54$ *because the formula is* $l \cdot w$ *and then just divided 54 by 6 and got 9 for w.*

Teacher: *So, that's the formula for area . . .*

Student: *Ohhh . . .*

Teacher: *. . . and you want perimeter instead, which is* $2l + 2w = 54$*. So since you know the length is 6, you can write [signals to student to start writing as he speaks]* $2(6) + 2w = 54$*. Right. Now what is* $2 \cdot 6$*?*

Student: *12?*

Teacher: *Right. And now we need to subtract the 12 from both sides of the equation [points to paper to indicate the student should write what he is suggesting]. And* $54 - 12$ *is . . . ?*

Student: *42?*

Teacher: *OK so if* $2w = 42$*, then how much is just one w?*

Student: *21?*

Teacher: *That's right! Make sure you write that all down. [Continues circulating room]*

What do we notice about how the teacher and student respond to one another? The student—for one reason or another—was looking for some sort of validation of their work. Work, it is worth mentioning, that was absolutely mathematically correct, albeit misplaced on this particular task. The teacher follows the student's inquiry with an open request for explanation, which could communicate the importance of process in the class. Once the student unveils their thinking, however, the teacher assumes a corrective stance and begins walking the student through the problem-solving process. The student seems to recognize their error in problem setup after the teacher informs them that "that's the formula for area," but is quickly cut off as the teacher proceeds to plow the *correct* solution path.

Let's think about what we can infer about their presumed roles and positions within that classroom. It is difficult to discern exactly how the student might presume their own role in the classroom based on this exchange, because frankly, we don't hear much from them. The teacher, however, appears to have assumed the role of Corrector-in-Chief. Which is an important and fitting role if our primary task as math teachers is to help students produce correct answers. It is clear that the teacher has situated himself as the arbiter of truth in this exchange—the master codex against which other participants might calibrate their own efforts. Now, we should be careful here not to completely demonize the familiar "sage on the stage" metaphor—for content expertise is an invaluable tool to facilitate the many roles teachers must navigate to promote a student-centered classroom. However, the consideration I am promoting here is regarding the impact the teacher is having on the student's sense of ownership in the content and classroom/course. Namely, how is the teacher's own positioning as the *knower* and *shower* affecting that of the student? Well, we can only infer based on what we see. The student was situated to only follow instructions and answer tightly close-ended calculations. Here are some reasonable conclusions from this exchange:

- *The teacher sets up the problem, and I solve it.*
- *I need to do this like the teacher.*
- *Calculations are the important part.*
- *The way I did it was wrong.*

Regardless, are these the messages that foster student ownership in mathematics? How might this student respond if we asked them *whose math is it?*

There was a clear decision-point for the teacher in this exchange after the student explained their thinking. Let's take a look at the same exchange again, this time highlighting the decision-point, along with some additional considerations on the part of the teacher and alternative responses. We will use the expert noticing framework (Jacobs et al., 2010) whereby we first attend to the details of the case, then interpret their meaning, and finally choose how to respond.

The perimeter of a rectangular swimming pool is 54 meters. The length of the pool is 6 meters. What is its width?

Student: *[raises their hand and signals the teacher over] Is this right?*

Teacher: *Can you tell me what you did?*

Student: *OK. Well, I wrote* $6 \cdot w = 54$ *because the formula is* $l \cdot w$ *and then just divided 54 by 6 and got 9 for w.*

Decision-Point

Expert Noticing: This student is correctly using variables to represent unknown quantities and is correctly solving for those quantities. However, this student set up the problem as if they were given the area of the pool of 54 square meters rather than the perimeter of 54 (linear) meters. There is a possibility that there is confusion around units (meters versus square meters), but it could have just been an oversight, and that also isn't the primary focus of this task. There is also a possibility that the student does not know the difference between area and perimeter, but that is not clear yet, so I will need to gauge more about this. Also, I want to be careful to honor the work the student has done and situate it as legitimate mathematics, though different than what the task is seeking. So, I want to use language that validates *their* process.

Teacher: *OK. I see what you did here, and I appreciate how you used variables to represent the unknown quantities. I heard how you talked through your problem-solving process and calculations, and it all sounded mathematically legitimate to me. So here's my question . . . How would you do this if the AREA of the pool was 54 square meters instead?*

Student: *[Silent for a moment while looking at their work, and the original problem.] The area is 54? Oh, OHH!!!*

Teacher: *Yup, there it is.*

Student: *Ahhh I did area instead of perimeter! [Starts erasing]*

Teacher: *Yeah you did and . . . [waves hands] No, no! Don't erase it! That's really great work for a different problem. Maybe we should even give it to the class next? Just write the new work for this problem underneath.*

Now what do we notice about how the teacher and student respond to one another? And what can we infer about their presumed roles and positions within that classroom? In contrast to the first exchange, this time the teacher led with validation and recognition of the student's legitimate mathematical thinking—which was not contrived. Then, we saw the teacher guide the student's thinking with a targeted question that held multifaceted value. Asking the student about area provided the teacher insight into whether the student recognized the difference between area and perimeter (one of the early content wonderings), as well as served as a prompt to trigger the student's thinking around the actual *ask* of the task. The teacher did not jump into premature reteaching—which would have served as a rigor-reducing overscaffold in this case.

Further, the teacher communicated confidence in the student's own recognition of what adjustments needed to be made, which could reinforce the student's sense of ownership and efficacy in mathematics. Finally, the teacher made very clear that the student should not *undo* their original work by erasing it. This final validating move of the student's thinking could only continue to perpetuate the message that their contributions matter and their mathematical thinking is worthy. So then, perhaps some reasonable conclusions from this second exchange might include the following:

- *The teacher is here to guide me but not do the work for me.*
- *Sometimes I need the teacher, and sometimes I don't.*
- *Calculations are important but so is correctly setting up a problem.*
- *The way I did it was right but for a different problem.*

Regardless, these contrasting messages could serve to foster greater student ownership in mathematics. How might this student now respond if we asked, *whose math is it?*

Our decisions in the classroom, our choices during planning, and the way we respond to students all have the propensity to greatly affect how students see themselves as mathematicians. We have the power to contribute to or detract from our students' sense of agency and mathematical ownership—all of which contributes to their ever-dynamic identities. We need to act with care, and we need to act with intention if we are to use our powers for good. Thus is the intent of this book. How can we structure our courses, classrooms, and ourselves toward this end of promoting mathematical ownership in our students?

How to Use This Book

This book is rooted in teacher clarity and split into two parts, both presented through the context of mathematics education: determining success criteria and operationalizing success criteria. The first part, Determining Success Criteria, is intended to help teachers clearly define success in mathematics in a way that is productive for their students. We will also look at relevant research and best practices, which is the focus of Chapter 1. The second part, Operationalizing Success Criteria, is intended to help teachers provide opportunities for students to build their success and ownership in mathematics in whole-class, peer-to-peer, and individual settings through the development of social and sociomathematical norms, collaborative learning experiences, and self-regulated learning.

Sociomathematical norms: norms that are specific to a *mathematics* learning community and regulate the community's communication about and participation with the subject of *mathematics.*

Chapter 2 will explore the teacher's role in developing classwide social and sociomathematical norms that underpin the mathematical culture of their classrooms. It will also discuss how to leverage the clarity gained in Chapter 1 to explicitly develop, maintain, and leverage social norms with social learning intentions. We will see that sociomathematical norms develop in any learning community whether we intend them to or not, for better or worse—so we ought to consider shaping them with intention. Chapter 2 will further illustrate how to communicate and model the existence of *choice* in mathematics, as well as how to use discursive positioning moves to situate our students as problem-solvers with agency. The mantra for mathematical ownership at the whole-class level in this chapter is *everybody's doing it*.

Chapter 3 will discuss how to reinforce student ownership by structuring peer interactions and collaboration and will make the case for investing in collaboration as a space for students to begin taking ownership of their learning. Importantly, this chapter will recognize that students need to be primed in order to ensure that group work is indeed productive. Everything from grouping strategies to setting up and launching tasks will be covered to this end. This chapter also serves as a hub for various collaborative strategies and protocols suitable for the mathematics classroom. The mantra for mathematical ownership among students at the peer-to-peer level in this chapter is *we're doing it*.

Chapter 4 homes in on supporting individual students by promoting metacognition and self-regulated learning—essential components of ownership. It will delineate the self-directive process of self-regulation into its individual components and discuss how to scaffold students toward increased motivation by targeting each for development. This includes teaching students how to become more independent learners and study. Finally, it will demonstrate the importance of feedback and student self-assessment in self-regulated learning. The mantra for mathematical ownership for students individually in this chapter is *I'm doing it.*

The book closes with a review of the student-facing mantras of this book and their implications, as well as provides some teacher-facing mantras to guide classroom policies and decision making. Implementation is as much about mindset as it is about action. Building student ownership of mathematics requires both a plan *and* a sense of direction. I aim to ensure this book provides both. The intent of this closing section is to facilitate a sense of ownership in the reader and communicate that *You can do it.*

Each chapter will begin with its own overarching learning intention and set of specific success criteria to ground your learning by communicating our goals. Success criteria will have additional callouts throughout each chapter to model *signaling*, an aspect of teacher clarity discussed in the next chapter that helps guide learning by providing additional structure. Each chapter will conclude with reflection questions, to help you make personal connections to your own practices and mathematical experiences, as teacher clarity also encompasses understanding ourselves. Speaking of clarity—let's start there.