Preface

Starting in the early 1990s, with the support of the National Science Foundation (NSF), four groups of scholars, researchers, and teachers began to investigate alternative modes of assessment in mathematics under the common name of Balanced Assessment in Mathematics. These groups, at the University of California at Berkeley, Michigan State University, the Shell Mathematics Center at the University of Nottingham, and the Harvard Graduate School of Education, collaborated and competed over the next decade in evolving a variety of approaches to the problem of helping teachers and schools assess the effectiveness of their instruction in mathematics.

At Harvard, we worked on devising a way to formulate the content and methods of mathematics in a way that teachers and students would be comfortable with. We worked closely with teachers in classroom settings to be sure that the abstractions of the academic world translated into workable and, indeed, beneficial classroom products—products that had a coherence across the grades, across the various content domains of mathematics, and across the diverse interests of students and teachers.

The core of our formulation of the field of mathematics started from the recognition that each of the thousands of spoken languages on our planet structures language in the form of NOUN PHRASE–VERB PHRASE. Such universality implies something quite fundamental about the way we perceive and talk about the world. We seem to want to describe our environment and the events that occur in it in terms of OBJECTS (noun phrases) and ACTIONS (verb phrases). Does this observation about language shed any light on the way we might think about mathematics?

We can formulate the subject of mathematics in terms of mathematical objects (e.g., numbers, shapes, patterns) and mathematical actions (e.g., addition, multiplication, reflection, scaling) that are carried out on or by them. We have found that this formulation helps teachers put their own knowledge of the subject of mathematics into a perspective that allows them to communicate more effectively and more coherently with their students. Lest you fear that this formulation provides yet another thing to learn and understand, we show how the National Council of Teachers of Mathematic's (NCTM, 2006) *Curriculum Focal Points* fit directly and easily into this framework.

We have also paid particular attention to the complex nature of problem solving. Solving a problem is not a unitary task. Some problems make strong demands on a student's ability to formulate a problem mathematically, while other problems make strong technical demands on a student's ability to compute but little demand on the understanding necessary to know what to compute. Still other problems require the student to make inferences based on the results of their formulations and computations. Collapsing all of these dimensions of solving a problem into a single grade does little to help the teacher understand where a student's strengths and weaknesses might lie.

We hope that this book succeeds in conveying our excitement for balanced assessment to teachers and students of mathematics and makes clear the intertwined nature of instruction and assessment.