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 Visit the companion website at

 http://resources.corwin.com/problemsolving3-5

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**Note From the Publisher:** The authors have provided video and web content throughout the book that is available to you through QR (quick response) codes. To read a QR code, you must have a smartphone or tablet with a camera. We recommend that you download a QR code reader app that is made specifically for your phone or tablet brand.

Videos may also be accessed at http://resources.corwin.com/problemsolving3-5

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