CHAPTER 2 Setting Goals and Selecting Tasks



Before you can begin to work on the five practices, you must first set a goal for student learning and select a task that aligns with your goal. Smith and Stein (2018) have described this as *Practice 0*—a necessary step in which teachers must engage as they begin to plan a lesson that will feature a whole class discussion. As they explain,

To have a productive mathematical discussion, teachers must first establish a clear and specific goal with respect to the mathematics to be learned and then select a high-level mathematical task. This is not to say that all tasks that are selected and used in the classroom must be high level, but rather that productive discussions that highlight key mathematical ideas are unlikely to occur if the task on which students are working requires limited thinking and reasoning. (Smith & Stein, 2018, p. 27)

In this chapter, we first unpack what is involved in setting goals and selecting tasks and illustrate what this practice looks like in an authentic high school classroom. We then explore challenges that teachers face in engaging in this practice and provide an opportunity for you to explore setting a goal and selecting a task in your own teaching practice.

Part One: Unpacking the Practice: Setting Goals and Selecting Tasks

What does it take to engage in this practice? This practice requires first specifying the learning goal for the lesson and then identifying a high-level task that aligns with the learning goal. Figure 2.1 highlights the components of this practice along with key questions to guide the process of setting goals and selecting a task.

Figure 2.1 • Key questions that support the practice of setting a goal and selecting a task

WHAT IT TAKES	KEY QUESTIONS	
Specifying the learning goal	Does the goal focus on what students will learn about mathematics (as opposed to what they will do)?	
	Does your task provide students with the opportunity to think, reason, and problem solve?	
Identifying a high-level task that aligns with the goal	What resources will you provide students to ensure that all students can access the task?	
	What will you take as evidence that students have met the goal through their work on this task?	

In the sections that follow, we provide an illustration of this practice drawing on a lesson taught by Cori Moran in her Transition to College Math class, a course that prepares eleventh- and twelfth-grade students for college and career pathways that are not STEM intensive. As you read the description of what Ms. Moran thinks about and articulates while planning her lesson, consider how her attention to the key questions influences her planning.

Specifying the Learning Goal

Your first step in planning a lesson is specifying the goal(s). Consider Goals A and B for each of the mathematical ideas targeted in Figure 2.2. How are the goals the same and how are they different? Do you think the differences matter?

For each of the mathematical ideas targeted in Figure 2.2, the goal listed in Column A is considered a *performance goal*. Performance goals indicate what students will be able to do as a result of engaging in a lesson. By contrast, each of the goals listed in Column B is a *learning goal*. The learning goals explicitly state what students will understand about mathematics as a result of engaging in a particular lesson. The learning goal needs to be stated with sufficient specificity such that it can guide your decision making during the lesson (e.g., what task to select

TARGETED IDEA	GOAL A	GOAL B
Exponential Functions Algebra	Students will be able to identify a function in the form of $f(x) = ab^x$ as an exponential function and use the equation to find missing values.	Students will understand that an exponential relationship of the form $f(x) = ab^x$ has a rate of change that varies by a constant multiplicative factor. As the function's input values increase by 1 the output value changes by a constant factor.
Similarity Geometry	Students will be able to identify similar figures.	Students will understand that two figures are similar if their corresponding angles are equal and the ratios of the lengths of their corresponding sides are equal.
Conditional Probability Statistics and Probability	Students will be able to apply the formula for conditional probability. $P(A B) = \frac{P(A \cap B)}{P(B)}$	Students will understand that conditional probability refers to the probability of one event (A) occurring given that another event (B) has already occurred.

Figure 2.2 • Different goals for learning specific mathematical ideas

for students to work on, what questions to ask students as they work on the task, which solutions to have presented during the whole class discussion). According to Hiebert and his colleagues (2007),

Without explicit learning goals, it is difficult to know what counts as evidence of students' learning, how students' learning can be linked to particular instructional activities, and how to revise instruction to facilitate students' learning more effectively. Formulating clear, explicit learning goals sets the stage for everything else. (p. 51)

In general, "the better the goals, the better our instructional decisions can be, and the greater the opportunity for improved student learning" (Mills, 2014, p. 2).

According to Hunt and Stein (in press), "too often, we define what mathematics we wish students to come to 'know' as performance, or what students will 'do,' absent the understandings that underlay their behaviors." If we want students to learn mathematics with understanding, we need to specify what exactly it is we expect them to understand about mathematics as a result of engaging in a lesson. Hence, goals you set for a lesson should focus on what is to be learned, not solely on performance.

Ms. Moran and her eleventh- and twelfth-grade students were working on a unit on polynomial functions. They had previously discussed what constitutes a term and defined a polynomial function as a mathematical function that is the sum of a number of terms. Since students had previously worked with linear functions, they had established that a linear function was a polynomial function of degree 1. In this lesson, students would be exploring quadratic functions, a specific type of polynomial function of degree 2. As a result of the lesson, Ms. Moran wanted her students to understand that

- 1. Quadratic functions may be characterized by two-dimensional growth.
- 2. Quadratic functions, unlike linear functions, do not have a constant rate of change.
- 3. Quadratic growth can be expressed in both recursive and explicit forms.
- 4. Quadratic growth can be modeled in a variety of ways—with a diagram in which width and height increase linearly, with a table where the second difference is constant, with a graph where the shape of a function is a parabola, and with an equation of degree 2.

The level of specificity at which Ms. Moran articulated the learning goals for the lessons will help her in identifying an appropriate task for her students, and subsequently, it will help her in asking questions that will move students toward the goal and in determining the extent to which students have learned what was intended. As Ms. Moran commented,

One of the benefits of setting clear goals is everything you do is a jumping point off from there. Once I set the goal, I knew what task I wanted to use, what questions I might ask, what solutions I might anticipate, and how I wanted to sequence solutions. I even could get some connecting questions ready. I was able to connect everything to those goals.

Identifying a High-Level Task That Aligns With the Goal

Your next step in planning a lesson is to select a high-level task that aligns with the learning goal. High-level or cognitively challenging mathematical tasks engage students in reasoning and problem solving and are essential in supporting students' learning mathematics with understanding. By contrast, low-level tasks—tasks that can be solved by applying rules and procedures—require limited thinking or understanding of the underlying mathematical concepts. According to Boston and Wilhelm (2015), "if opportunities for high-level thinking and reasoning are not embedded in instructional tasks, these opportunities rarely materialize during mathematics lessons" (p. 24). In addition, research provides evidence that students who have the opportunity to engage in high-level tasks on a regular basis show greater learning gains than students who engage primarily in low-level tasks during instruction (e.g., Boaler & Staples, 2008; Stein & Lane, 1996; Stigler & Hiebert, 2004).

TEACHING TAKEAWAY

Specificity is one of the keys to setting learning goals and finding appropriately aligned tasks. Tasks that provide the richest basis for productive discussions have been referred to as *doing-mathematics* tasks. Such tasks are nonalgorithmic— no solution path is suggested or implied by the task and students cannot solve them by the simple application of a known rule. Hence students must explore the task to determine what it is asking them to do and develop and implement a plan drawing on prior knowledge and experience in order to solve the task (Smith & Stein, 1998). These tasks provide students with the opportunity to engage in the problem-solving process—understand the problem, devise a plan, carry out the plan, and look back (Polya, 2014). Central to this process is the opportunity for students to wrestle with mathematical ideas and relationships that are inherently part of the task.

While the level of cognitive demand is a critical consideration in selecting a task worthy of discussion, there are other characteristics that you should also consider when selecting a task. Specifically, you need to consider the following: the number of ways that the task can be accessed and solved, the extent to which justification or explanation is required, the different ways the mathematics can be represented and connected, and opportunities to look for patterns, make conjectures, and form generalizations. These characteristics are a hallmark of rich mathematical tasks and help ensure that students will have the opportunity to engage in the mathematics practices/processes (e.g., make sense of problems and persevere in solving them, reason abstractly and quantitatively, construct viable arguments, use repeated reasoning) that are viewed as essential to developing mathematical proficiency. When sizing up the potential of a task, keep in mind the questions shown in Figure 2.3. These questions will help you in selecting rich tasks that will make engagement in key mathematical practices and processes possible.

Figure 2.3 • Questions to help you *size up* the richness of a task

- Are there multiple ways to enter the task and to show competence?
- Does the task require students to provide a justification or explanation?
- Does the task provide the opportunity to use and make connections between different representations of a mathematical idea?
- Does the task provide the opportunity to look for patterns, make conjectures, and/or form generalizations?

Answering "yes" to each of these questions does not guarantee that students will engage in the mathematical practices. However, the use of the five practices, together with doing-mathematics tasks that have the characteristics described, will help ensure that this will occur. So rather than thinking about separate process goals, such as the Standards for Mathematical Practice advocated for in the Common Core State

TEACHING TAKEAWAY

Doing-mathematics tasks, rather than procedural exercises, lend themselves to rich and productive mathematical discussion.

Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), we encourage you to consider characteristics of tasks that will provide your students with the opportunities to engage in such processes.

For her lesson, Ms. Moran selected the Staircase task, shown in Figure 2.4, for several reasons. The task showed growth in two dimensions (i.e., the staircase was getting both taller and wider) and it could be solved in different ways and expressed in both recursive and explicit forms. The image of the growing staircase would provide students with an opportunity to concretely explore how the function was growing and how the stage number was related to the number of squares in the staircase for that stage. Ms. Moran also liked the fact that the questions provided scaffolding for students. The first two questions provided students with the opportunity to see how the staircase was changing and what a bigger staircase would look like if the pattern continued. To answer the third question, students would need to begin to move beyond the concrete and consider what the tenth staircase would look like given what had occurred with the first five staircases. The final question required students to generalize their findings in order to determine a function that would describe relationship between the total number of small squares in any stage and the stage number.





This task can also be accessed at resources.corwin.com/5practices-highschool

To ensure that students would have access to the task, Ms. Moran planned to provide students with a cup containing two different colors of square tiles they could use to build staircases at different stages, extra paper, markers, and physical and digital graphing resources. She would leave it up to the students to decide which, if any of these resources, would be useful. In addition to these material resources, Ms. Moran also decided that she would provide "human" resources by having students work on the task in groups so that they would have others with whom to confer.

When asked what students would say, do, or produce that would provide evidence of their understanding of the goals in the lesson through their work on this task, Ms. Moran indicated that students would do the following:

- state the number of square tiles in each of the stages (e.g., stage 4 has 10 square tiles, stage 5 has 15 square tiles, stage 10 has 55 square tiles);
- recognize that the staircase is growing in two dimensions—that it was getting both wider and taller;
- notice that the rate of change is not constant so it could not be linear;
- relate the stage number to the number of squares in a stage by making more familiar shapes from the squares in a particular stage; and
- make the connection that because the staircase was growing in two dimensions that the equation will contain a x^2 term.

Cori Moran's Attention to Key Questions: Setting Goals and Selecting Tasks

During her initial stage of lesson planning, Ms. Moran paid careful attention to the key questions. First, in setting her goals for the lesson, she clearly articulated what it was she wanted students to learn about mathematics as a result of engaging in the task. The specificity with which she stated her goals made it possible to determine what students understood about these ideas and to formulate questions that would help move her students forward. In addition to wanting her students to write a function that would describe the relationship between the number of squares in a stage and the stage number, she also wanted to make sure they understood what makes a function quadratic and how to recognize a quadratic function in different representational forms.

Second, she identified a high-level *doing-mathematics* task that aligned with her goals. Students could not solve the Staircase task by applying a known rule or procedure because the task required students to look for the underlying structure of the pattern of growth and use the identified pattern to generalize to other stages in the pattern. Ms. Moran *sized up*

the task (see Figure 2.3) to ensure that it had several other important characteristics. Specifically, there were a number of approaches that students could use to enter the task (e.g., making a table showing the stage number and the number of squares in the stage, using square tiles to build specific stages, adding to or rearranging the square tiles in a stage to form a shape that could be more easily described) and the material and human resources that the teacher planned to provide would support their work. The task made it possible for students to use different representations build a model, make a table, create an equation, and "explain how you know" (question 4). Finally, the task asked students to generalize their findings by describing the relationship between the stage number and the number of squares in the stage. Hence through their work on the task, students could learn important mathematics and engage in key practices.

The Staircase task, along with the resources the teacher made available to students, allowed all students to enter the task at some level. Rather than differentiating instruction by providing different students with different tasks, she selected one task and met the needs of different learners by providing resources for students to consider and questions that would challenge learners at different levels.

Finally, Ms. Moran indicated some things that she expected students to say and do that would provide evidence students were making progress on the ideas she wanted them to learn. By considering this evidence in advance of the lesson, she was ready to pay close attention to students' work for indications that they were making progress in their understanding.

Through her careful attention to setting goals and selecting a task, Ms. Moran's planning was off to a productive start and she was ready to engage in the five practices. In the next chapter we will continue to investigate her planning process as she anticipates what she thinks her students will do when presented with the task and how she will respond. We now turn our attention to the challenges that teachers face in setting goals and selecting tasks.

Part Two: Challenges Teachers Face: Setting Goals and Selecting Tasks

As we described in the chapter opening, setting goals and selecting tasks is foundational to orchestrating productive discussions. Setting goals and selecting tasks, however, is not without its challenges. In this section, we focus on four specific challenges associated with this practice, shown in Figure 2.5, that we have identified from our work with teachers. Figure 2.5 • Challenges associated with the practice of setting goals and selecting tasks

CHALLENGE	DESCRIPTION
Identifying learning goals	Goal needs to focus on what students will learn as a result of engaging in the task, not on what students will do. Clarity on goals sets the stage for everything else!
Identifying a doing- mathematics task	While <i>doing-mathematics</i> tasks provide the greatest opportunities for student learning, they are not readily available in some textbooks. Teachers may need to adapt an existing task, find a task in another resource, or create a task.
Ensuring alignment between task and goals	Even with learning goals specified, teachers may select a task that does not allow students to make progress on those particular goals.
Launching a task to ensure student access	Teachers need to provide access to the context and the mathematics in the launch but not so much that the mathematical demands are reduced and key ideas are <i>given away</i> .

Identifying Learning Goals

Identifying learning goals is a challenging but critical first step in planning any lesson. It is challenging because we often focus on what students are going to be able to do as a result of engaging in a lesson, not on what they are going to learn about mathematics.

Consider, for example, the lesson that Michael Moore, one of our three featured teachers, was planning for his geometry students. When Mr. Moore initially planned the lesson, he indicated that he wanted students to "identify similar triangles and use their properties to solve problems." He selected the Floodlight Shadows task, shown in Figure 2.6, to accomplish this goal. Given time constraints, Mr. Moore planned to focus on questions 1 and 2 in one class period and reserve work on question 3 for the following day.

During a discussion about the lesson Mr. Moore was planning around the first two questions in the task, his colleagues pointed out that his goal was fairly broad. One of the teachers asked Mr. Moore if he could explain what he would be looking and listening for as students worked that would help him know if the students were meeting his goal. Mr. Moore first focused on the creation of the mathematical model of the geometric situation, since the creation of the model was what would allow students to start discussing similar triangles. Through the discussion with his colleagues, it became clear to Mr. Moore that the creation of a scale model or drawing was a critical step in moving students toward an exploration of similarity and should be a part of his goals. He explained that he wanted students to "see how those ideas actually show up in a model." As the conversation with his peers continued, Mr. Moore was able to clarify in detail what understandings of mathematics students would need to solve

Figure 2.6 • The Floodlight Shadows task



Source: This task is part of a carefully engineered lesson "Deducing Relationships: Floodlight Shadows" from the Mathematics Assessment Project, which can be downloaded for free from mathshell.org

resources

A link to this task can be accessed at resources.corwin.com/5practices-highschool

the Floodlight Shadows task. As a result, he was able to provide more specificity regarding what he wanted students to learn from the lesson and what he might be looking for in their work. Specifically, as a result of engaging in the lesson, Mr. Moore wanted his students to understand that

- creating mathematical models of geometric situations is helpful in identifying key features that can be used to make mathematical arguments;
- 2. similar triangles are related in two ways: their corresponding angles are congruent, and the lengths of their corresponding sides are related by a common scale factor; and
- 3. justifying that two triangles are similar means making arguments about the ways in which corresponding angles are congruent, and corresponding sides are scaled using ideas such as corresponding parts of triangles and scale factor.

Mr. Moore also noted that the task had many features that would provide opportunities for his students to engage in the standards for mathematical practices (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), including creating viable arguments using geometry properties as well as modeling with mathematics. With new clarity regarding what he wanted students to understand, Mr. Moore was now ready to anticipate how students would engage with the task and prepare questions that would help him illuminate what his students understood about these ideas as they worked through the lesson. Why does this level of specificity matter? It matters because with this level of specificity, the teacher will be able to better listen for key student ideas and ask questions that draw students' attention to the meaningful features of the diagram without giving them a step-by-step recipe for determining similarity. When Mr. Moore interacts with his students as they work on the task, he can ask them where they see different triangles in the visual model. He can press them to consider how the triangles might be related, what features are the same across the triangles, and what features are different. He can also ask questions that aim to move students from judging the triangles as similar through visual inspection to creating a mathematical argument for that similarity using ideas about angle measures, side measures, and scale factor. By paying attention to how students will use the resources in the task and their visual diagrams to build rigorous similarity arguments, Mr. Moore will be able to determine what his students understand about the meaning of similarity in triangles and the relationships between similarity and dilations. This information will then help him in planning subsequent lessons in the unit to continue to develop the broader themes about dilations. (See Hunt and Stein [in press] for a description of three interconnected phases—defining a goal, unpacking the mathematics and refining the goal, and relating goals to pedagogy-that teachers can use individually or collaboratively to create and refine goals for student learning.)

While posting goals in the classroom has become commonplace, you might wonder if a goal at the level of depth and detail as the one Mr. Moore created is one that should be shared with students. You will often find that the mathematical goals you created to guide your decision making during a lesson need to be different from the mathematical goal you share with your students. The mathematical goal you choose to share with your students should focus their learning and communicate what is expected of them within the lesson without providing too much direction or guidance.

For the Floodlight Shadows task, for example, you might share the following goal with students: *Today, we will make mathematical arguments about relationships between triangles in a contextual situation.* A goal such as this explicitly connects to important aspects of prior knowledge by signaling that students will be doing work with triangles. The goal stops short of specifying that students will be determining similarity, so it leaves the question of what relationship exists between the triangles students will create as an open question for exploration. The goal also calls attention to making mathematical arguments a key mathematical practice at the heart of the lesson. On the whole, this goal focuses students on important content and processes that will be useful to them in the lesson, while still leaving significant exploratory territory for students to work within as they do the mathematics through engagement in the task.

Identifying a Doing-Mathematics Task

While doing-mathematics tasks provide the optimal vehicle for whole class discussions, not all curricular materials are replete with such tasks. Traditional textbooks tend to feature more procedural tasks that provide limited opportunities for reasoning and problem solving. While such resources do include *word problems*, they are often solved using procedures that have been previously introduced and modeled and require limited thinking. While standards-based texts (Senk & Thompson, 2003) contain some procedural tasks, they also include high-level tasks that promote reasoning and problem solving.

While there are many ways to provide high-level tasks to your students, such as finding tasks in other resources, adapting existing tasks, or creating your own task, using a curriculum that contains a wide variety of doing-mathematics tasks is the optimal choice for a number of reasons. First, it's simply a little bit less work for you as a teacher! But perhaps more importantly, standards-based curricula have been carefully crafted to develop important mathematical ideas with focus, rigor, and coherence. These curricula typically have thoughtful author teams made up of mathematicians and mathematics educators, undergo significant field testing and revision, and represent the current best thinking on ways to organize students' mathematical learning. The sampling of high school curricula, shown in Figure 2.7, contains a high number of doing-mathematics tasks; those marked with an asterisk have their genesis in the set of National Science Foundation–funded curricula of the mid-1990s.

Figure 2.7 • A sampling of high school curricula that contains high-level mathematics tasks

Commercially Available Curricula

- Contemporary Mathematics in Context (CORE-Plus) (https://www.mheducation.com/prek-12/program/core-plus-mathematics-20152015/ MKTSP-QRE07M0.html)*
- Interactive Mathematics Program (IMP)

(http://activatelearning.com/interactive-mathematics-program-imp/)*

Open Source/Free Curricula

- Open Up Resources High School Mathematics/Illustrative Mathematics (https://openupresources.org/math-curriculum/high-school/)
- MathMontana (formerly SIMMS Integrated Mathematics) (https://mathmontana.org)*
- Mathematics Vision Project
 (https://mathematicsvisionproject.org/)

If you are using a resource that does not include high-level tasks, what should you do? First and foremost, we encourage you to consider advocating for the adoption of a curricular resource with high-level tasks with your district during your next adoption cycle (or before!). In the meantime, in this section we explore three possible options for finding high-level tasks—adapt an existing task, find a task in another resource, or create your own task.

Adapting an Existing Task

Some textbook tasks either give students too much information and leave little for students to figure out on their own or ask students to simply recall and apply a previously learned rule. In such cases, you may want to consider adapting the task so as to provide more thinking opportunities for students. This section provides two examples of adapted tasks: the Binomial Expansion task shown in Figure 2.8 and the Heights of Dinosaurs task shown in Figure 2.9. Binomial Expansion is an example of a task with a fairly simple modification that provides opportunities for additional student reasoning. Heights of Dinosaurs is an example of a more extensive modification to remove unnecessary scaffolding and hence provide more opportunities for students to determine what to do and how to do it.

Grace Sullivan was working on a unit on quadratics, and she wanted her students to understand and be able to use a variety of representations and procedures for expanding and simplifying polynomials. She wanted students to feel comfortable using different methods, but also to understand why those methods worked and to be able to make connections between them. When she found the original Binomial Expansion task (shown in Figure 2.8), she was pleased to see that her textbook resource was also focusing on multiple representations. She decided to make a simple modification to the task—to ask students to compare two of the methods and relate them to the broader context of how the methods are helpful in determining the product.

Ms. Sullivan's modification of the task was relatively simple and did not require much time to complete. By asking students to compare two of the methods, she would be able to get additional information on how students were making sense of the different methods and of the broader concepts of multiplying binomials. For example, comparing the distributive property and FOIL methods would afford students the opportunity to see that FOIL isn't simply a memorized shortcut, but a double application of the distributive property. The algebra tile method would allow students to see that the binomial factors can be represented as the dimensions of a rectangle and the area can be connected to the products in any of the other three methods. The vertical method provides an important connection to whole-number multiplication, which brings conceptual understanding TEACHING Takeaway

If your textbook does not provide worthwhile tasks for discussion, you have options: adapt, find, or create your own!

Figure 2.8	٠	The	Binomial	Expansion	task
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Source: Holliday et al. (2005), Algebra I, p. 455, #2.



This modified task can also be accessed at resources.corwin.com/5practices-highschool

and meaning to the other methods. By adding in a simple modification to the task, the opportunities to strengthen the understanding and build procedural fluency from conceptual understanding are heightened.

When George Jacobson saw the Heights of Dinosaurs task in the instructional materials for his Algebra II with Statistics course (Figure 2.9), he noticed that it walked students through a series of steps. He wondered if his students might get caught up in the mechanical details and lose the bigger mathematical idea—namely, in what ways does a data distribution help us understand what's typical in a data set? Mr. Jacobson decided to modify the task so that his students would be able to do more thinking and reasoning. Toward that end, he wanted to pose the question, "How tall was a compy?" at the start of the investigation rather than saving it until the end. This way, the question would motivate students' mathematical investigations.

Figure 2.9 • Heights of Dinosaurs task



Compsognathus (dinosaur image) created by Nobu Tamura, http://spinops.blogspot.com. CC BY 2.5 https://creativecommons.org/licenses/by/2.5/

The heights were determined by studying the fossil remains of the compys.

Height (cm)	Number of Compys	Relative Frequency
26	1	0.002
27	5	0.008
28	12	0.018
29	22	0.033
30	40	0.061
31	60	0.091
32	90	0.136
33	100	0.152
34	100	0.152
35	90	0.136
36	60	0.091
37	40	0.061
38	22	0.033
39	12	0.018
40	5	0.008
41	1	0.002
Total	660	1.000*

*The values above add to 1.002; this discrepancy is due to rounding.

Exercises 1–8



(Continued)

Figure 2.9 (Continued)

- 1. What does the relative frequency of 0.136 mean for the height of 32 cm?
- 2. What is the width of each bar? What does the height of the bar represent?
- 3. What is the area of the bar that represents the relative frequency for compys with a height of 32 cm?
- 4. The mean of the distribution of compy heights is 33.5 cm, and the standard deviation is 2.56 cm. Interpret the mean and standard deviation in this context.
- 5. Mark the mean on the graph, and mark *one* deviation above and below the mean.
 - a. Approximately what percent of the values in this data set are within one standard deviation of the mean
 - (i.e., between 33.5 cm 2.56 cm = 30.94 cm and 33.5 cm + 2.56 cm = 36.06 cm)?
 - b. Approximately what percent of the values in this data set are within *two* standard deviations of the mean?
- 6. Draw a smooth curve that comes reasonably close to passing through the midpoints of the tops of the bars in the histogram. Describe the shape of the distribution.
- 7. Shade the area of the histogram that represents the proportion of heights that are within one standard deviation of the mean.
- 8. Based on our analysis, how would you answer the question, "How tall was a compy?"

Source: Lesson 9: Using a Curve to Model a Data Distribution, Eureka Math. Copyright © 2016 by Great Minds. Allrights reserved.

Mr. Jacobson also felt that the task could be launched in a more engaging way. When students typically think of dinosaurs, they think of beasts far larger than humans. Yet the compy data show that these dinosaurs were closer in size to mid-sized birds. Mr. Jacobson remembered that these dinosaurs were featured in a memorable scene in the film *The Lost World: Jurassic Park II* and searched online for a video clip that might help to motivate the task. Inspired by the Three-Act Task format popularized by Dan Meyer (http://threeacts .mrmeyer.com) and Robert Kaplinsky (http://robertkaplinsky.com), the teacher thought that asking students to make observations, determine an inquiry of focus (i.e., how tall was a typical compy?), and decide what data they might need to have to answer that question would provide them with ownership and increase student motivation and engagement, rather than just handing students the compy height data set and having them get started.

Drawing on these ideas, Mr. Jacobson developed the Compy Attack! lesson. His slides and some notes about them are shown in Figure 2.10. Figure 2.10 • Slides Mr. Jacobson prepared for launching the Compy Attack! lesson



Compsognathus (dinosaur image) created by Nobu Tamura, http://spinops.blogspot.com. CC BY 2.5 https://creativecommons.org/licenses/by/2.5/

(Continued)

Figure 2.10 (Continued)



SETTING GOALS AND SELECTING TASKS



Source: Adapted from Lesson 9: Using a Curve to Model a Data Distribution, Eureka Math. Copyright © 2016 by Great Minds. All rights reserved. Act 1 image source: THE LOST WORLD: JURASSIC PARK Compy Attack (Youtube) via Stan Winston School. Act 3 image *Source:* Dinosaur attack in Jurassic Park 2 (Youtube) via Richard Stenger.

resources

This modified task can also be accessed at

resources.corwin.com/5practices-highschool

In comparing the original task to the modified task, you will see that Mr. Jacobson made the following changes:

The question about the height of a typical compy ends Act 1 and motivates the investigation, rather than being the end point of the work.

Rather than stepping students through a series of statistics calculations, the task asks them to use any statistical tools they choose to make their argument.

Some additional motivation is provided for the launch and summary of the task using video clips.

• Students have multiple pathways available to them to identify what is typical and to justify their thinking.

By making these changes, Mr. Jacobson transformed a low-level task into a doing-mathematics task. In addition, his modified task has the additional characteristics we previously discussed—it can be entered and solved in several different ways, it requires students to use and make connections between different representations, it asks students to explain their thinking, and it asks students to generalize. (See Arbaugh, Smith, Boyle, Stylianides, & Steele, 2019; Boyle & Kaiser, 2017; and Smith & Stein, 2018 for more insight on how to modify tasks.)

Finding a Task in Another Resource

You can find high-level doing-mathematics tasks in many print and electronic resources. The challenge is to find a task that meets your mathematical goals, is accessible to your students, has the potential to advance their learning beyond their current level, and fits with the content and flow of your curriculum.

For example, Michael Moore went to the Mathematics Assessment Project website (https://www.map.mathshell.org) to see if he could find a task that would help him reach the goals he had set for the lesson. He was familiar with the site and knew that it contained interesting tasks as well as complete lessons. He selected the Floodlight Shadows lesson, which included a detailed lesson plan. While he did not follow the plan as a script, it helped him think through how he wanted to orchestrate his lesson. By contrast, Kendra Nelson, who you will meet in the example that follows, selected the Triple Trouble task from a set of related lessons (Algebra II Lesson Set: Building Polynomial Functions) that the district had purchased. The advantage of this resource is that it contained a set of eight related tasks that as a collection had the potential to develop students' conceptual understanding of polynomial functions. So in addition to using the Triple Trouble task, Ms. Nelson could also consider the tasks that followed it in the sequence to continue the exploration of polynomial functions. (See Appendix A for a list of web and print resources that may be helpful to you in finding doing-mathematics tasks.)

Ms. Nelson's Algebra II class had been working with families of functions for most of the quarter. They had explored and compared linear, quadratic, and exponential functions, and Ms. Nelson was looking for a good bridge into the unit introducing polynomial functions. Unfortunately, most of what she found in her textbook strongly emphasized symbolic manipulation and did not provide students multiple entry points. She knew based on their previous work that her students brought a diverse set of skills to the table when it came to functions. Some students were extremely adept at graphing and making sense of a function visually, others focused on tables and numeric values, and several gravitated toward algebraic notation and manipulation. Ms. Nelson felt that the unit needed a broader variety of entry points that focused on making meaning of functions. As a result of engaging in this first lesson, Ms. Nelson wanted her students to understand the following:

• The key features of polynomial functions—*x*-intercepts/ roots, *y*-intercepts, local and absolute minima and maxima, and discontinuities—can be identified using multiple representations (equation, table, graph).

- The product of two or more polynomial functions is a polynomial function. The product function will have the same *x*-intercepts as the original functions.
- Two or more polynomial functions can be combined using addition, subtraction, or multiplication using their graphs or tables, because given two functions f(x) and g(x) and their associated numerical values, the sum, difference, or product of the two functions will be on the graph and the table as a specific value, x, and the values of the sum, difference, or product of the two functions, f(x) and g(x).

Ms. Nelson's district math leader suggested that she take a look through an ancillary print resource the district had purchased, which included sets of related lessons (including lesson guides) for some high school units. The Triple Trouble task (shown in Figure 2.11) was immediately compelling to Ms. Nelson. The task brought together a number of important elements that she expected would help students make a strong transition to thinking about polynomial functions. It began with two function families that were very familiar to her students—linear and quadratic. The task culminated in finding a new function, which would be cubic, that was the product of these two functions. Rather than jumping right into the symbolic work, the task asked students to evaluate four claims that fictional students made about the characteristics of h(x).





(Continued)

Figure 2.11 (Continued)

Consider the two functions shown in the graph. Let $h(x) = f(x) \cdot g(x)$.

- David, Theresa, Manuel, and Joy are working in a group together to determine the key characteristics of h(x). They each make a prediction. Decide whether you agree or disagree with each student's prediction. Use mathematics to justify your position.
 - a. David: h(x) will be a parabola.
 - b. Theresa: h(x) will have a y-intercept at (0, 12).
 - c. Manuel: h(x) will have negative y-values over the interval $-2 \le x \le -1$.
 - d. Joy: h(x) will have three x-intercepts.
- 2. Sketch a graph of h(x) on the coordinate plane. Then identify key characteristics of the graph (zeros, y-intercept, maximum/minimum values, end behavior) and explain how each characteristic results from key characteristics of f(x) and g(x).
- 3. Determine the equation of h(x). Justify your answer in terms of f(x) and g(x).

Source: Institute for Learning (2017). Lesson guides and student workbooks available at ifl.pitt.edu.

In working through the task on her own, Ms. Nelson identified a number of strategies that could be used to evaluate each of the four claims. Moreover, approaches that worked well for some claims were not as effective in evaluating other claims. For example, the first claim could be effectively argued from a conceptual understanding of functions, noting that a first-order (linear) function multiplied by a second-order (quadratic) function would result in a third-order (cubic) function, and therefore David's claim is wrong. The third claim about negative values between x = -2 and x = -1, in contrast, could be effectively justified by examining the range of the two functions over that interval and making a numerical argument about the products of negative and positive numbers, and therefore Manuel's claim is correct.

The second and third questions in the task would allow groups to draw together their observations from evaluating the claims into a more comprehensive picture of the function h(x). The key features could be identified using a variety of the approaches that were useful in evaluating the claims in question 1. Finally, question 3 asks students to create the equation for h(x). By positioning the more mechanical symbolic work at the end of the task, students would be more likely to make connections between the conceptual understanding of functions, the key features of the function as represented in the sketched graph, and the symbolic representation of the function. This would provide an excellent entry point into the broader unit on polynomials.

A note of caution here. There are many good tasks available electronically and in print (including those described in Appendix A) that could be interesting explorations for students. However, students develop an understanding of mathematical ideas as the result of engaging in sequences of tasks, not single tasks that are disconnected from what preceded or followed them. As Hiebert and his colleagues (1997) noted,

[The] teacher's role in selecting tasks goes well beyond choosing good individual tasks, one after another. Teachers need to select sequences of tasks so that, over time, students' experiences add up to something important. Teachers need to consider the residue left behind by sets of tasks, not just individual tasks. (p. 31)

Creating a Task

If you cannot find a task you want to use, you may decide to create one yourself. This was the case for Denise Hansen. Ms. Hansen's students were beginning a unit on probability. She wanted her students to understand the notion of sample space as the set of all possible outcomes and the Fundamental Principle of Counting as a way to determine the number of outcomes. This would lay the groundwork for the investigation of more advanced probability concepts in subsequent lessons. Specifically, through their work on the task, Ms. Hansen wanted students to understand that if one event could happen *m* different ways and another event could happen *n* different ways, then the number of outcomes for both events together would be $m \times n$.

While Ms. Hansen's textbook started the unit with students rolling dice and flipping coins, she decided that she wanted to select a context that students would immediately relate to. Toward this end, she created the Lunch Options task shown in Figure 2.12, which was based on the lunch menu in the school cafeteria that day. By starting with question 1, which students could solve by listing all the outcomes and counting, and ending with question 5 that could not be easily modeled or listed, Ms. Hansen was pressing students to generalize their findings so that determining the number of You Pick Two options would be a simple matter of multiplying $7 \times 19 \times 3$.

Figure 2.12 • Lunch Options task

Lunch Options

The Hot Entrée Combo in the cafeteria includes your choice of an entrée with 1 side and a drink for \$6.50. Today's selections are shown below.

Entrée	Sides	Drinks
• Baked ziti with meat	Steamed broccoli	• Milk
 Baked ziti with veggies 	 Caesar salad 	• Water
	Garlic breadsticks	• Apple juice
	Chicken soup	

(Continued)

Figure 2.12 (Continued)

- 1. How many different lunch combinations are there? Explain how you figured it out and how you know you have them all.
- 2. Suppose the combo also included a choice of 2 desserts: a peanut butter cookie or a chocolate brownie. How many different lunch combinations would there be then? How do you know?
- 3. Compare your answers to the first two questions. How are they different? Explain.
- 4. How many different combinations do you think you will have if you now add a choice of a peppermint or spearmint candy to your combo? Explain.
- 5. Panera Bread[™] offers You Pick Two[™], which includes a choice of 2 half portions from the following list: soup, macaroni, salad, and sandwich. You decide to have soup and a sandwich. There are 7 different soups and 19 different sandwiches. In addition, you can select a side of an apple, chips, or a baguette. Drawing on your work on the first four questions, determine how many different combinations you could put together.



This task can also be accessed at resources.corwin.com/5practices-highschool

Because it cannot be solved by using a known rule or procedure, thus requiring students to reason and problem solve, Lunch Options is a doing-mathematics task. The task has other characteristics that make it a good choice for the lesson as previously described:

- It can be entered and solved in several different ways (e.g., make a list of possible combinations, model the situation, make a drawing, describe in words what is happening).
- It asks students to explain their thinking, not just provide an answer.
- The task provides the opportunity for students to make connections between a visual representation (tree diagram) and a numerical representation $(m \times n)$.

In order to answer question 5, students need to look back at their work on the first four questions, identify a pattern, and then use the pattern to determine the number of You Pick Two combinations.

While creating tasks is certainly an option, it is a challenging endeavor! If you decide to create a task yourself, we encourage you to ask others to review and solve the task so that you can identify any possible pitfalls before you give it to students. This is exactly what Ms. Hansen did with the Lunch Options task!

Ensuring Alignment Between Task and Goals

Another challenge teachers often face is making sure that there is alignment between the task and goals—that is, ensuring that the task they have selected as the basis for instruction provides students with the opportunity to explore the mathematical ideas that are targeted during the lesson. As we described in Part One of this chapter, Ms. Moran was clear about what she wanted her students to learn about mathematics (not just what they would do) and she selected a high-level doing-mathematics task that was consistent with her learning goals. This combination of a high-level task and a clear learning goal is optimal for providing students with the opportunity to learn mathematics with understanding.

But what would it look like to have a mismatch, and what implications does this have for instruction and learning? Suppose, for example, you are teaching a lesson on exponential functions. You decide that you want students to understand that an exponential relationship, of the form $f(x) = ab^x$, has a rate of change that varies by a constant multiplicative factor; as the function's input values increase by 1 the output value changes by a constant factor (Goal B in the first row of Figure 2.2). You select the Evaluate the Functions task shown in Figure 2.13, which asks students to find the value of y give a value for x. This basic naked numbers task does not provide students the opportunity to develop an understanding of situations that can be represented by exponential functions, what "b" and "x" mean in context, or what it means for the output to change by a constant factor as the input increases by 1 unit. As a result, students may end up with correct or incorrect solutions but it would not be clear (to them or to you) whether they had a clear understanding of the targeted mathematical idea. In this situation, you have established a *learning* goal but have paired it with a low-level task that requires students to only apply their knowledge of exponents and the order of operations. If your goal had been for students to find values for y given values for a, b, and x (Goal A in the first row of Figure 2.2), then the low-level task would be a good match for this goal.

Figure 2.13 • Evaluate the Functions task

Evaluate each of the functions.				
1.	$f(x) = 4^x \text{ at } x = \frac{1}{2}$			
2.	$f(x) = \left(\frac{1}{2}\right)^x \text{ at } x = 5$			
3.	$f(x) = (10)2^x$ at $x = -2$			
4.	$f(x) = \left(\frac{1}{5}\right) \left(\frac{1}{3}\right)^x \text{ at } x = 3$			
5.	$f(x) = (-5)5^x$ at $x = 4$			

If you want students to understand that an exponential function has a rate of change that varies by a constant multiplicative factor (as the function's input values increase by 1 the output value changes by a constant factor), then a better choice would be a task such as the one shown in Figure 2.14. The Family Tree task provides a context for making sense of and modeling an exponential growth situation and is a good introductory task. Students are free to create drawings, graphs, or tables in order to explore growth from one generation to the previous one. In solving the task, students have the opportunity to discover that the number of "great" grandparents doubles every time you go back one additional generation and that the number of the of the generation (2^x) . Through work on a problem such as the Family Tree, students can come to understand what an exponential function is and what each of the variables represents.

Figure 2.14 • The Family Tree task

Family Tree

Ramona was examining the family tree her mother had created using Ancestry.com, a website you can use to explore your genealogy. When examining a family tree, the branches are many. Ramona is generation "now." One generation ago, her 2 parents were born. Two generations ago, her 4 grandparents were born. Three generations ago, her greatgrandparents were born. Four generations ago, her great-greatgrandparents were born. Every time she went back another generation, another "great" was added!

- 1. How many of Ramona's great-grandparents were born 3 generations ago?
- How many of Ramona's "great" grandparents were born 10 generations ago?
- Explain how Ramona could figure out how many "great" grandparents were born in any number of generations ago.
- 4. Is the number of "great" grandparents in any generation a function of the generation? Why or why not? How do you know?

online This task can also be accessed at resources.corwin.com/5practices-highschool

Ensuring alignment between your goals and task is essential and the foundation on which to begin to engage in the five practices. If you find that your task does not fit your goals, consider the ways in which you can modify the task in order to provide more opportunities for students to think and reason as we described previously.

Launching a Task to Ensure Student Access

Launching or setting up a task refers to what the teacher does prior to having students begin work on a task. While it is not uncommon to see teachers hand out a task, ask a student volunteer to read the task aloud, and then tell students what they are expected to produce as a result of the work on the task, research suggests that attention to the way in which the task is launched can lead to a more successful discussion at the end of the lesson. Jackson and her colleagues (Jackson, Shahan, Gibbons, & Cobb, 2012) describe the benefits of an effective launch:

Students are much more likely to be able to get started solving a complex task, thereby enabling the teacher to attend to students' thinking and plan for a concluding whole-class discussion. This, in turn, increases the chances that all students will be supported to learn significant mathematics as they solve and discuss the task. (p. 28)

So what constitutes an effective launch? In Analyzing the Work of Teaching 2.1, you will explore Ms. Moran's launch of the Staircase task. We invite you to engage in the analysis of a video clip and consider the questions posed before you read our analysis. [NOTE: While the launch of a task occurs during instruction, it is planned for prior to instruction. Rather than describing what Ms. Moran intended to do, we decided to take you into her classroom so that you could see for yourself!]



Launching a Task—Analysis

Ms. Moran began the lesson by showing students the images of the first four stages of the staircase on the interactive whiteboard and asking students what patterns they noticed. There are two aspects of this introduction that are important. First, by not immediately giving students access to the entire task, students' attention was focused only on the images and not on the questions they would subsequently have to answer. Second, the teacher cautioned students to "just take a second. Don't say anything right away." This gave students time to look closely at the images and make observations about the staircases and how they were growing and changing before ideas were made public. The focused yet openended nature of what students were asked to do provided all students with access to the task (everyone would be able to notice something) and their responses would help Ms. Moran determine whether students could identify key features of the growing figure that would help them in ultimately solving the task.

Two key ideas came up during the discussion of what students noticed that were critical in making sense of the problem. First, Nyleah explained that for stage 5, it would be "like 5 going across [width], 5 going up and down [height], and then 5 going down the stairs." She later generalized by saying, "Each stage is the amount of how many should be on the bottom and up and down and going down the stairs, too." At the teacher's request, Nyleah went up to the whiteboard to show exactly what she was referring to using stage 4 (see Figure 2.15). Nyleah's explanation made clear the parts of the staircase she was referring to.

Figure 2.15 • Nyleah's illustration of how the stage number related to the number of blocks in the stage



Next Elijah explained that the stage number was the same as the number of blocks on the bottom row. Araia, building on Elijah's explanation, further clarified: "The number of the stage is how many blocks you're adding. So stage 3, you add 3 blocks. Stage 4, you add 4 blocks ... "This comment connected the bottom row of a stage that Elijah referred to as the number of blocks that were added from the previous stage.

The observations made by students could ultimately help them in recognizing that the growth was not linear—since the amount added on to each successive stage was not the same—and the staircases were growing in two dimensions, with both the width and height increasing at each new stage. Ms. Moran concluded the launch by describing the materials students would have access to and telling students that they were now going to explain and describe the patterns they noticed in their groups. The launch made it clear to Ms. Moran that students were attending to features of the staircases that would help them in reaching the goals of the lesson.

Was the time spent launching the task time well spent? We could argue that it was essential to ensuring that students were able to make sense of how the staircases were growing and thus positioning students to be ready to generalize the growth they were observing. Too often, students are given a task and do not understand some aspect of it. When this occurs, the teacher then ends up moving from one group to the next answering questions that could have been clarified with a more comprehensive launch.

Jackson and her colleagues (2012) list "four crucial aspects to keep in mind when setting up complex tasks to support all students' learning" (p. 26):

- 1. Key Contextual Features of the Task
- 2. Key Mathematical Ideas of the Task
- 3. Development of Common Language
- 4. Maintaining the Cognitive Demand

Ms. Moran's launch embodied many of the features described by Jackson and colleagues. The teacher (1) made sure that students could describe a staircase at a particular stage in terms of its shape and the number of squares that were been added to the previous staircase; (2) made sure that students recognized that a staircase had two dimensions (although they did not state this explicitly); (3) let students use common language to describe the dimensions (i.e., bottom, up and down) but made sure that these terms were understood by relating this verbal description to the visual of the staircase; and (4) ensured that the cognitive demand of the task was maintained by not suggesting a pathway to follow or giving away too much information.

TEACHING TAKEAWAY

A comprehensive launch ensures all students understand the entirety of the task, so that time is not spent later answering clarifying questions from groups or individuals. While Ms. Moran's launch was appropriate given the task she had selected, other tasks might require different types of launches. For example, in launching the Floodlight Shadows task, Mr. Moore wanted to make sure that students had a basic understanding of shadows and how they changed when the distance from the light source changed. He started the lesson by darkening the room, standing on his desk with a high-power flashlight and shining the light on a student volunteer, and asking questions about what was happening and why. This, he felt, would position students to engage in the task. As Mr. Moore explained,

The most important part of understanding this task is the shadows and why shadows are formed. I'll really focus on when shadows are formed—we're talking about light and light that's being blocked out, and really, hopefully planting that seed a little bit into their mind of the role that the light plays in their diagram.

Mr. Jacobson decided to launch the Compy Attack! lesson by showing students a short clip of a movie and asking students what they noticed and wondered about from their viewing of the video. He felt that this would motivate students, get them interested in solving the task, and raise authentic questions, some of which they would ultimately answer. While launches can take many forms, as a teacher you need to make sure that students understand the context of the task and are clear on what they are expected to do.

Perhaps the most challenging part of launching a task is making sure that you do enough to ensure that students understand the context and what they are being asked to do but not so much that there is nothing left for students to figure out. For example, in the Lunch Options task (Figure 2.12), you would not want to show students how to create a tree diagram or give them the Fundamental Principle of Counting. Either of these actions would lower the demand of the task by providing a strategy for students to use and limit their opportunity to figure out what to do and how.

Conclusion

In this chapter, we have discussed the importance of setting clear goals for student learning and selecting a task that is aligned with the goal, and we have described what is involved in this practice and the challenges associated with it. Our experience tells us that if you do not take the time to seriously consider Practice 0 as a first step in carefully planning your lesson, the remainder of the practices will be built on a shaky foundation.

Ms. Moran's work in setting a goal and selecting a task provided a concrete example of a teacher who thoughtfully and thoroughly engaged in this

practice. Engaging in this practice in a deep and meaningful way does not happen overnight. It takes time and practice. As she said, "I think it's really important to really plan, plan, plan... It's important work. And I think it just gets better and better as you work with these practices."

Mr. Moore's efforts to determine what students would learn during the lesson and to ensure that the goals and tasks align made salient the challenges that teachers can face and overcome when engaging in this practice. Working with colleagues helped him make progress on Practice 0.



In the next chapter, we explore the first of the five practices: *anticipating*. Here, we will return to Ms. Moran's lesson and consider what it takes to engage in this practice and the challenges it presents.

Linking the Five Practices to Your Own Instruction <

SETTING GOALS AND SELECTING TASKS

Identify a mathematical idea that you will be teaching sometime in the next few weeks. Working alone or with your colleagues, do the following:

- 1. Determine what it is you want students to learn about mathematics as a result of engaging in the lesson. Be as specific as possible. It is okay to indicate what students will do during the lesson, but do not stop there!
- 2. Select a high-level cognitively demanding *doing-mathematics* task that is aligned with your goals. Make sure that there are different ways to enter and engage with the task. Identify resources that are likely to help students as they work on the task.
- 3. Identify what students will say and do that indicates that they are meeting the goals you have established.
- 4. Plan a launch that takes into account the four crucial aspects identified by Jackson and her colleagues.

Reflect on your planning so far. How does it differ from how you have previously thought about goals and tasks? In what ways do you think the differences will matter instructionally?

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