WHAT YOUR COLLEAGUES ARE SAYING . . .

"While reading this book and watching the videos, I found myself wishing that I had read it when I was a classroom teacher. The authors give readers a window into a learning community of educators as they apply their new sense-making about mathematics to their teaching. While the academic content is mathematics, we are guided through the creation of a community where each learner's sense of self and agency is strengthened around their own learning."

Janice E. Jackson
Former Deputy Superintendent, Boston Public Schools
Former Deputy Assistant Secretary, Office of Elementary and Secondary Education, U.S. Department of Education Newark, CA

"This book is a must-read for math educators across K–12 who want to foster curiosity, meaning making, and collective mathematical agency in our nation's diverse classrooms. The authors and their teacher-leader partners provide multiple resources that illustrate equitable community-focused instruction where students take charge of and practice essential mathematical skills including representing, conjecturing, and generalizing. Powerful!"

Julia Maria Aguirre
Professor of Education, University of Washington Tacoma
Tacoma, WA

"This resource provides images of teaching that promote student voice and agency in mathematics classrooms. The authors' creative approach—with video clips, transcripts, and commentaries from collaborating teachers and critical friends—will engage teachers, coaches, and leaders across the professional continuum. The images, mathematical representations, questions, and activities will transform your thinking and practice and ultimately elevate student participation and learning."

Kathryn B. Chval
Dean and Professor, Mathematics Education, University of Illinois Chicago
Chicago, IL

"I sum up *Interweaving Equitable Participation and Deep Mathematics* as: Deep Mathematics Content + Community Participation that leads toward equitable engagement, growth, and outcomes. Teachers and leaders can engage in deep professional learning throughout the sections in this book and will reflect on their practices and students' learning."

Robert Q. Berry, III
Dean, College of Education, The University of Arizona
Tucson, AZ

"Central to this book is the valuing and foregrounding of children's mathematical thinking. Through classroom videos and supporting reflection prompts and activities, the authors show the interplay of providing rich mathematical experiences and focusing on equitable participation. This results in a powerful resource for teachers, teacher educators, and anyone interested in a mathematics education that honors all children's ideas."

Marta Civil
Professor and Roy F. Graesser Chair, The University of Arizona
Tucson, AZ

"In this exciting book, Russell and Schifter invite us to join their professional learning community to explore teaching that interweaves a commitment to equity and rigorous mathematics. Each chapter offers vivid examples of teachers and students engaged in rich mathematical tasks and deep collaborative conversations in their classrooms. It really is a privilege to reflect alongside these authors. I can't wait to share this book with my teacher education students."

Sandra Crespo
Professor and Associate Chair of Graduate Education,
Department of Teacher Education,
Michigan State University
East Lansing, MI

"Together with collaborating teachers and critical friends, Russell and Schifter take us into inspirational and real classrooms, revealing the complexities and insights that come with taking students' intellects seriously and nurturing equitable classroom communities. This book invites rich reflection and entices you to move with urgency and eagerness to develop your mathematics teaching and make real your commitments to racial equity. You will return to it again and again."

Elham Kazemi
Professor, Mathematics Education, University of Washington
Seattle, WA

"Russell and Schifter have created a remarkable resource for teachers to gather around! The authors have expertly combined rich videos with insightful analysis and thought-provoking questions. Readers will have the best sort of collaborative, empowering opportunities to grow both a deep understanding of mathematics teaching and learning and their capacity to provide equitable instruction to all students. This book is a gift!"

Tracy Johnston Zager
Math Coach
Author, *Becoming the Math Teacher You Wish You'd Had*
Coauthor, the *Building Fact Fluency* toolkits
Portland, ME

Interweaving Equitable Participation and Deep Mathematics

Building Community in the Elementary Classroom

Susan Jo Russell

and

Deborah Schifter

with

Collaborating Teachers: Quayisha Clarke, Emmanuel Fairley-Pittman, Natasha Gordon, Jeff Parks, Isabel Schooler, and Michelle Sirois

and

Critical Friends: Cynthia Ballenger, Virginia Bastable, Yi Law Chan, Marta Garcia, Lynne Godfrey, Hetal Patel, and Darlene Ratliff

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Contents

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Note From the Publisher: The authors have provided video and web content throughout the book that is available to you through QR (quick response) codes. To read a QR code, you must have a smartphone or tablet with a camera. We recommend that you download a QR code reader app that is made specifically for your phone or tablet brand.

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Preface

The idea that sparked the project that eventually resulted in this book came from our colleague, Elizabeth Sweeney. “You have to capture these teachers and their students on video,” she told us one summer afternoon over lunch by the seashore. By “these teachers,” she meant graduates of the Master’s program she worked in who were now teaching in public, neighborhood schools in Boston, Massachusetts. Liz knew we had often filmed classroom examples in previous projects, including in her own classroom twenty years earlier when she was a fifth-grade teacher. She believed, as we do, that, when the focus is on students’ sense-making, video can be an important tool to slow down and show what learning and teaching can look like—with real students and real teachers in real time. Liz was now mentoring teachers who had a strong commitment to equitable participation in schools with significant populations of Black and brown students—groups that have been historically marginalized in mathematics (Joseph & Alston, 2018; Ladson-Billings, 2006, 2007). Liz—and we—saw an opportunity for a partnership in which a program of rigorous mathematics would be intertwined with a focus on supporting students’ identity and agency as mathematics learners in a classroom mathematics community. We and the teachers would learn together, analyzing video and written documentation of the students’ work. We believed from the beginning that this collaboration would lead to rich and thought-provoking professional development resources.

We first came to know Liz, a long-time employee of Boston Public Schools, as a teacher-participant in a professional development program we were leading. Reflecting on her view of mathematics teaching when she entered the program, Liz said, “I never had the belief that math was about thinking, reasoning, or making sense. It was just learning the rules. There was no *self* in math.” Once Liz began to see mathematics as a realm of exploration, she discovered she loved the changes her growing engagement with mathematics brought about in her preparation, her teaching, and her classroom, and she saw her fifth graders respond enthusiastically as she engaged them in thinking deeply about mathematical ideas. Liz taught fifth grade for many years, then went on to become a math coach and, later, a citywide leader of elementary mathematics. Upon retirement from Boston Public Schools, Liz joined the faculty of the Boston Teacher Residency (BTR).

BTR is a joint initiative of the Boston Plan for Excellence and Boston Public Schools that combines a year of residency in a school setting working with an experienced teacher and targeted master's-level coursework. As BTR math faculty, Liz provided experiences like those in her own professional development that had changed her beliefs and practices about mathematics and mathematics teaching. During their residency, the teachers she was supporting engaged in doing mathematics for themselves and were learning how to establish a classroom community based on students sharing their thinking and building concepts together about important mathematical ideas. Once they began teaching in their own classrooms, Liz continued as their mentor through BTR. Following up on Liz's suggestion, as plans for a joint project developed, we were able to obtain support both from BTR and from TERC, a nonprofit organization in Cambridge, Massachusetts, that focuses on mathematics and science education.

Liz proposed that the collaborating teachers would use lesson sequences that we had recently published (Russell et al., 2017). These eight lesson sequences investigate sets of related generalizations about the basic operations: addition, subtraction, multiplication, and division. Written for Grades 1 to 5, each sequence consists of about twenty 20-minute sessions. The sequences are designed to be used in addition to the regular math program, much like Number Talks or other such routines.

The videos at the heart of this book come from the classrooms of six teachers, graduates of BTR, who were teaching from these lesson sequences. For two years, we visited the teachers' schools, video recorded lessons, met with the teachers individually and as a group, and exchanged writing about what we and they saw in the classroom. Susan Jo was behind the camera, while Deborah kept a running record of the class session. You'll notice Deborah, and sometimes also Liz, taking notes in some of the videos. After the recording portion of the project was complete, we asked each teacher to review the videos from their classroom and interviewed them about their decisions, their questions, and what they learned. We also selected video clips to discuss with our own study groups and to present at workshops and conferences.

Here is a little information about each of the Collaborating Teachers you will be meeting in this book:

Quayisha Clarke is an experienced educator with a decade of teaching within the Boston Public School system. She holds National Board Certification and currently serves as an Instructional Coach at Dudley Street Neighborhood Charter School. In addition to her professional commitments, she is dedicated to furthering her knowledge and is pursuing a PhD specializing in Human Development and Learning at Lesley University. With her wealth of experience and ongoing academic pursuits, Ms. Clarke is committed to delivering high-quality education and support to students, teachers, and educational institutions. Ms. Clarke was a Grade 2 teacher during the project.

Emmanuel Fairley-Pittman is an Inclusive Education Coach supporting teachers and schools in Boston to plan for and implement inclusive practices. As a National Board certified educator having taught third, fourth, and fifth grade at the Grew School in Hyde Park for eight years before this role, he is committed to providing access for all students so that they have meaningful academic and social experiences in school. He attributes his commitment to the equitable teaching of mathematics in part to his own teachers growing up, specifically Ms. Langston and Mrs. Hunter. These teachers believed in his abilities and provided the necessary tools and support for him to grow into his identity as a mathematician. Mr. Fairley-Pittman was a Grades 3–4 teacher during the project.

Natasha Gordon's collaboration with exceptional math coaches enabled her to cultivate curiosity and foster independence in her first-grade students. Her passion lies in ensuring equal opportunities for all learners, fostering success, and embracing diverse ideas. Currently serving as an equitable literacy coach, she strives to continue to create and uplift inclusive educational environments. Ms. Gordon was a Grade 1 teacher during the project.

Jeff Parks is a facilitator of professional learning for the Telescope Network and math instructional coach at the Mather Elementary School. Before this, Mr. Parks taught third grade in Dorchester at the Mather Elementary and Everett Elementary for a combined 10 years. He has served in these schools as the Boston Union Rep, teacher leader, and tech coordinator. He has also worked with Boston Teacher Residency as a graduate coach and facilitator. His commitment to group work and collaborative problem solving in mathematics continues to reinforce his belief that all students can do math at the highest level. Mr. Parks was a Grade 3 teacher during the project. He will be returning to the classroom next year and is excited to teach math again with his new fourth-grade students.

Isabel Schooler taught first grade for six years, then worked as an elementary special education teacher for a year. In her current role as a math interventionist and coach, she draws on her experience with outstanding math coaches, finds inspiration in the thinking of young mathematicians, and remains dedicated to equity in mathematics. She looks forward to returning to the classroom this fall! Ms. Schooler was a Grade 1 teacher during the project.

Michelle Sirois has been teaching for eleven years. After graduating from the Boston Teacher Residency Program, she taught in Boston Public Schools for eight years. She currently is a Math Specialist for Grades 3–5 in Milford Public Schools. Throughout her years in the classroom, she has focused on building communities of mathematical learners where students notice patterns, ask questions, and play with numbers and ideas. She truly believes that children are sense-makers. Ms. Sirois was a Grade 4 teacher during the project.

As we, the authors, began to tease out the interwoven themes to be addressed in this book, we sought out the perspectives of other educators to view video from the classrooms of these six Collaborating Teachers through the lenses of their different experiences and backgrounds. We are both native English speakers of European descent, and we wanted to be informed by other educators from a variety of backgrounds who, throughout their careers, have focused on issues of identity, race, language, and inclusion. These educators, whom we call our "Critical Friends," provide analysis and raise their own questions throughout the book. Below is a quick introduction to each of our Critical Friends:

Cynthia (Cindy) Ballenger received a doctorate in Applied Linguistics from Boston University. She has worked as a teacher in public schools for many years and as a professor and program director at Tufts University. A central focus of her teaching has been the role of talk and culture in children's learning and participation, as in her books *Teaching Other People's Children: Literacy and Learning in a Bilingual Classroom* and *Puzzling Moments, Teachable Moments: Practicing Teacher Research in Urban Classrooms*.

Virginia Bastable is one of the authors of the *Developing Mathematical Ideas* professional development curriculum published by NCTM and two books on mathematical reasoning about the operations: *Connecting Arithmetic to Algebra* and *But Why Does It Work: Mathematical Argument in the Elementary Classroom*. She also contributed to the third edition of *Investigations in Number, Data and Space*. Before retiring, Dr. Bastable was the Director of the Mathematics Leadership Program and its precursor, SummerMath for Teachers, at Mt. Holyoke College. She started her education career as a high school math teacher, a job she enjoyed for more than twenty years. Her current work includes facilitating online academic year courses and summer institutes for the Master of Arts in Teaching Mathematics at Mount Holyoke.

Yi Law Chan is a New York City-based school leader where she is prioritizing social-emotional wellness and student-centered instruction. She brings to her current work over twenty years of experience as a former classroom teacher, math coach, assistant principal, and math specialist. In these roles, she organized and facilitated professional learning communities in examining the impact of equity-based teaching practices and teacher content knowledge on student learning.

Marta Garcia is an elementary mathematics specialist/coach with over thirty years of experience in teaching, coaching, and facilitating professional learning. She currently works as a mathematics coach with a variety of school districts across the U.S., teaches graduate courses focusing on mathematics leadership, and is the co-host of a virtual professional network of math coaches. She received the Presidential Award for Excellence in Mathematics and Science and the

NCCTM Rankin Award for distinguished service in the area of mathematics teaching and learning. Much of her current work focuses on how to empower teachers in the development of equitable classrooms where the voices of students, including multilingual learners and those from marginalized groups, are lifted.

Lynne Godfrey designs and facilitates professional development with coaches, teachers, and administrators to develop and sustain ambitious, equitable learning communities in their schools. As an educator, she served as a classroom teacher of Grades 2–8, a math coach in the Boston Teacher Residency program and elsewhere, and a director of instruction. Her work with Bob Moses and the Algebra Project for over thirty years, both locally and nationally, has influenced and sustained her ongoing commitment to access and equity for all adults and children in mathematics.

Hetal Patel is a mathematics instructional leader in New York City where she facilitates professional learning for teachers and school-building leaders. Additionally, she was an assistant principal, a school-based mathematics coach, and a math intervention teacher. In prior years, she facilitated several of the *Developing Mathematical Ideas* modules at Mount Holyoke College and nationally. Her professional interests include using Japanese Lesson Study to build professional learning communities, learning more about equity and developing identity in math education, and cultivating curiosity and joy in mathematics teaching and learning.

Darlene Ratliff, a recent retiree from Boston Public Schools, worked with general (K–8) and special education populations as an administrator, special education teacher, math coach, and math facilitator. She has served on the Assessment Development Committee for the Massachusetts Department of Elementary and Secondary Education, taught at the Horace Mann School for the Deaf and Hard of Hearing, and is a nationally certified American Sign Language interpreter. She has facilitated professional development workshops locally and nationwide, including *Developing Mathematical Ideas* seminars and *Looking at Student Work* sessions. Through witnessing firsthand the productivity and potential of all students, she developed her mantra: “Regardless of differing abilities, all students have the potential to achieve.”

As you read this book and view the classroom videos, we invite you to be in conversation with many voices—with us, with the Collaborating Teachers, with our Critical Friends, with the many students you will observe shaping their mathematics identities—and, for the best experience in using this resource, to reflect about your own students with your own colleagues. Listening to and respecting all of these voices provides openings for honesty and persistence in efforts to create classroom communities that interweave rigorous mathematics and equitable participation.

Susan Jo Russell
Deborah Schifter
June 2024

Acknowledgments

Many people have contributed to the project that produced this book. Without the passion and insight brought to their teaching by the Collaborating Teachers, their willingness to have their lessons videotaped and to meet with us to reflect on students' work, and their writing included here, this book would not exist. We cannot thank enough Quayisha Clarke, Emmanuel Fairley-Pittman, Natasha Gordon, Jeff Parks, Isabel Schooler, and Michelle Sirois. Similarly, we are so grateful to our partners in reflection, our "Critical Friends"—Cindy Ballenger, Virginia Bastable, Yi Law Chan, Marta Garcia, Lynne Godfrey, Hetal Patel, and Darlene Ratliff—who met with us, viewed and reflected on the classroom videos, and spoke from their hearts with the intensity and determination they bring to all of their work. Short bios of the Collaborating Teachers and Critical Friends can be found in the Preface. We want to thank Liz Sweeney, who started us on the path that led to this book (see the Preface for that story), the Boston Teacher Residency for making Liz's involvement with the project possible, and particularly Julie Sloan, then Director of the BTR Early Career Teaching Network. Christine Connolly, a Boston Public Schools principal at the time of this project, worked with us to implement professional development for the teachers and was instrumental in arranging our visits, videotaping, and meetings with the teachers. We are grateful for the support of Linda Davenport, then Boston Public Schools Director of K–12 Mathematics and a long-time colleague, for her support as our collaborating department head for Boston Public Schools. And, throughout the project, members of the Professional Development Study Group, a cross-institutional forum in the Boston area that has met for many decades to share work in progress, provided critique and support as we shared video clips from our work. We thank the Education Research Collaborative at TERC, TERC President Laurie Brennan, and the TERC Board of Trustees for recognizing the potential of this collaborative opportunity and funding this project. And we are grateful to the associate editor and publisher at Corwin, Erin Null, for her belief in and support of this book.

Finally, this book would not exist without the many students in the classes we observed and videotaped. We learned from them, were surprised by them, and were, again and again, struck by the joy and seriousness with which they pursued their ideas, once given the opportunity to voice them. We are delighted to be able to share their work with readers of this book.

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About the Authors

Dr. Susan Jo Russell began her career in education as a K–3 classroom teacher and elementary mathematics coach. For the last four decades, she has been a senior researcher at TERC, a nonprofit organization that focuses on mathematics and science education, where she directed projects on children's mathematical understanding and on supporting teachers to learn more about mathematics and about children's mathematical thinking. She spearheaded the original development and second edition of the K–5 mathematics curriculum, *Investigations in Number, Data and Space,* and contributed to the launching of the Forum for Equity in Elementary Mathematics. In recent years, her research has centered on how students engage in early algebra, number and operations, and mathematical argument, and how to support teachers to engage all of their students in high-level mathematical reasoning.

Dr. Deborah Schifter has worked as an applied mathematician; has taught elementary, secondary, and college level mathematics; and, since 1985, has been a mathematics teacher educator and educational researcher at Mount Holyoke College and at the Education Development Center. She authored *Reconstructing Mathematics Education: Stories of Teachers Meeting the Challenge of Reform* and edited a two-volume anthology of teachers' writing, *What's Happening in Math*

Class? for which she received the American Educational Research Association Professional Service Award in recognition of an outstanding contribution relating research to practice. She was a writer for *The Mathematical Education of Teachers* as well as the second and third editions of the K–5 curriculum, *Investigations in Number, Data, and Space.* Her recent research has focused on students' mathematical generalizations and how students use a variety of representations to explain why such generalizations are true.

Deborah Schifter and **Susan Jo Russell** have collaborated on many projects. With Virginia Bastable and a group of collaborating teachers, they produced the professional development series, *Developing Mathematical Ideas,* which is designed to help teachers think through the major ideas of K–8 mathematics and examine how children develop those ideas. With Virginia and another group of teachers, they developed *Connecting Arithmetic to Algebra*, which shows how investigating the behavior of the operations can move K–6 students forward. *But Why Does It Work? Mathematical Argument in the Elementary Grades*, coauthored by Susan Jo, Deborah, and others, is a resource for teachers who want to learn how to integrate mathematical argument into their instruction. (See the Resources section for further information.)

Introduction

What Is a Mathematics Community?

What does a mathematics community look like in an elementary classroom? How do we—teachers, coaches, administrators, all of us who support student learning—engage young mathematicians in deep and challenging mathematical content? How do we ensure that every student contributes a voice to this community, including students who have been historically marginalized in mathematics, students who have not believed they have mathematical ideas that are important to share, or who, when they have tried to express their ideas, have not been heard? These are the core questions this book seeks to address.

This book focuses on the interweaving of two commitments to children: a commitment to teaching deep and challenging mathematics and a commitment to equitable participation for all students in the classroom community. Without the opportunity for students to engage in significant mathematical content, a focus on equity is empty. If there are systems in place to ensure that every student speaks, but the math content is superficial and devoid of sense-making, we are not preparing students to become mathematical thinkers. On the other hand, if we attend exclusively to the rigor and depth of the mathematics, a few students may dominate, and what's perceived to be a correct and complete response from one or two students may stop continued discourse. Without attention to how each student engages with the content, the depth of the mathematics makes no difference for too many students.

In the intersection of deep mathematics and equitable participation, we have classrooms in which the *mathematics* content is significant, and the *community* enables each student to grow in understanding through their participation. In these classrooms, each student is assumed to have mathematical ideas, and it's the work of all of us to learn to listen for them. But as classroom teachers, coaches, instructional leaders, and others responsible for students' learning, how do we build and sustain such a community?

Four Aspects of a Mathematics Community

For two years, we, the authors of this book, undertook a research project in which we visited and videotaped lessons in the classrooms of six public elementary school teachers who were working to create classroom communities in which all students were engaged in serious content. Throughout this book we will refer to them as our Collaborating Teachers. We documented lessons, collected student work and teachers' writing, and reflected on these lessons with the teachers in order to uncover key ideas that characterized their evolving mathematics communities. As you move through the book, you yourself will interact with the videos and teachers' writing, observing and reflecting on students' thinking as well as the actions and thoughts of the teachers as they build mathematical communities in their classrooms. You will also hear the reflections and observations of our Critical Friends, who brought their own experiences to bear on what they saw in these classrooms. We'll explain more about this in a bit.

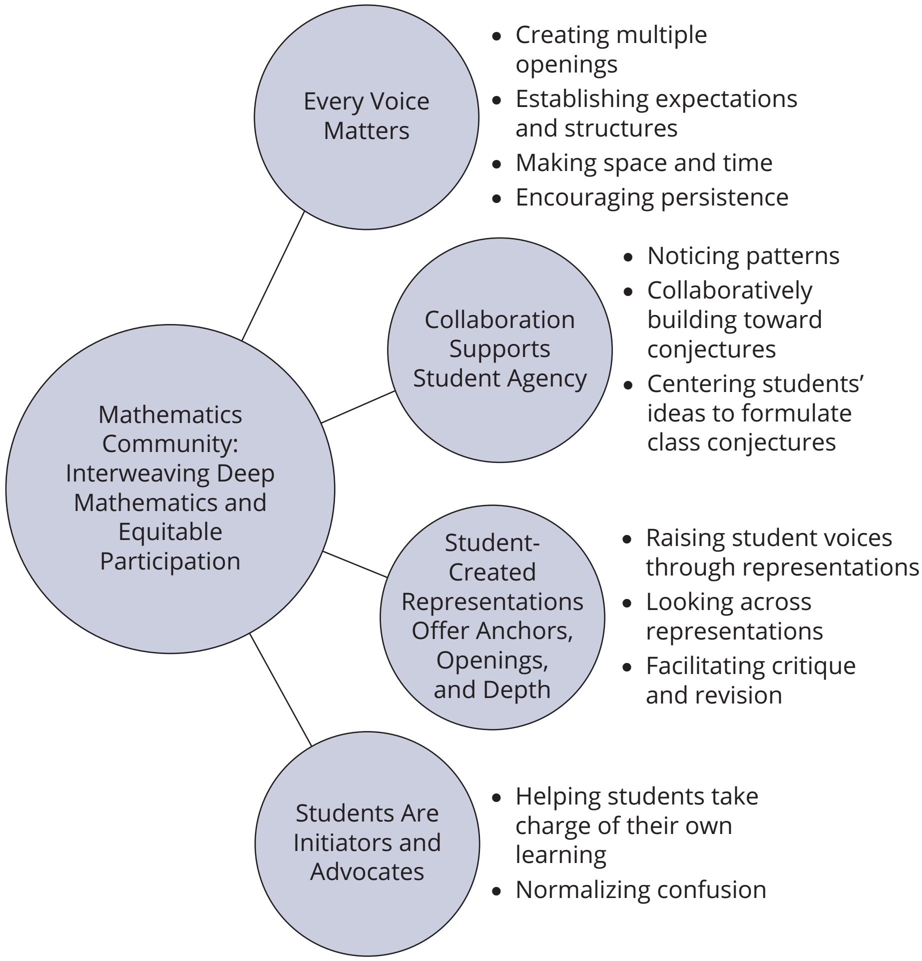
For our Collaborating Teachers, community was characterized by four main ideas: *every voice matters; collaboration supports student agency; student-created representations offer anchors, openings, and depth;* and *students are initiators and advocates for their own learning.* This book is therefore structured in four parts, each focusing on issues related to one of these ideas. While these four aspects of community interact and strengthen each other, we separate them in order to dig more deeply into what it takes to build each facet of a mathematics community (see Figure Intro.1).

Part One: Every voice matters.

In a mathematics community focused on student thinking, teachers establish classrooms in which students learn to take on the responsibility of sharing their ideas and attending to those of their classmates. The chapters in Part One focus on the importance of trusting students to take on challenging ideas as they develop their *identities* as doers of mathematics—how they view their own confidence and competence in approaching mathematical problems and questions. The videos illustrate how students are offered many modes of participation as they are beginning to develop their *mathematical agency*, that is, their inclination and ability to rely on their own reasoning. Both students who are eager to contribute their ideas and students who need more time and support to articulate their thoughts are included in the classroom discourse. Teachers and students listen intently to discern the sense in each student's thinking.

The culture of a mathematics community, as seen in the videos, is established through the teachers' sustained and intentional efforts. In Part One, our collaborating teachers discuss the norms they set and the tools they provide in the beginning of the year to help students understand what it means to

Figure Intro.1 • Four Aspects of a Mathematics Community



contribute to a mathematics discussion. Maintaining that culture requires the efforts of all participants—students and teacher—throughout the year.

Part Two: Collaboration supports student agency.

Mathematics is about complex ideas. Even in the primary grades, if given the opportunity and the tools, students engage in deep and abstract mathematical ideas. Unlike the image of the solitary, brilliant student working alone to solve every problem, much of mathematics requires collaboration, or what Aguirre et al. (2024) call "collective mathematical agency": "Classrooms of students can exhibit *collective mathematical agency* when teachers and their students act together to solve problems, working from the shared belief that viable strategies can be developed and solutions can be found. Different students can contribute different elements to this collective agency" (p. 17).

The chapters in Part Two focus on how students build ideas together as they notice patterns and articulate conjectures based on those patterns. The videos and commentary illustrate several related ideas: how offering partially formed or not yet well-articulated ideas for consideration is productive and, in fact, critical to the work of the community; how the practices of questioning and revising contribute to the community's ideas; and how it's everyone's responsibility to try to understand and build on each other's thinking.

Part Three: Student-created representations offer anchors, openings, and depth.

Student-created representations in the form of pictures, diagrams, models, and story contexts ground student ideas, encourage interaction, and draw more students into mathematical thinking. In contrast to the common view that students must be weaned off the use of diagrams and manipulatives to engage in abstract realms, understanding mathematics deepens for all students when they make connections across different forms of representation. Further, creating and explaining their own pictures, models, and story contexts are key parts of students' expressions of their own mathematical identities. You will see in Part Three how students are passionate and engaged as they use their own representations to explain their thinking, field questions from other students, and revise their representations to make their ideas clearer.

The chapters in Part Three present students' representations of pairs of related equations or story problems that illustrate a given generalization. By referring to their representations, students create meaning for symbols. Representations can also be key to mathematical argument, demonstrating *why* a procedure works or *why* a generalization is true.

The videos show students working together to interpret a set of representations chosen by the teacher from those created by the class. Commentary considers such issues as the connections made by looking at different representations, how a teacher selects which representations to share, and the value of choosing some representations that may need revision.

Part Four: Students are initiators and advocates for their own learning.

As students collaborate to build ideas together, they learn to pose their own questions and challenges. Students might ask questions such as the following: Will this pattern work with a different kind of number? What if I try it with very large numbers or very small numbers? Will it work the same for odds and evens? In this way, students become initiators of their own learning.

Students also advocate for themselves by identifying and articulating their confusion. When students engage in discussion with the expectation that they can understand, they learn to pause the discussion if they don't understand. Such pauses are recognized as a contribution to the discourse, allowing all community members to dig deeply and to explain their ideas more clearly. In these ways, students become advocates for their own learning while enhancing the class's collective agency.

Part Four illustrates how each student takes on the responsibility to be an active member of the mathematics community *and* an advocate for their own learning. The chapters include discussion of such themes as strategies a teacher might employ to encourage students' mathematical curiosity while maintaining the coherence of the lesson and what both the teacher and students gain by taking time to address a student's expressed confusion.

The Math

As we examine mathematics community, we give equal weight to "mathematics" and "community." While there are many discussions in the field of education that focus on equitable participation and the development of community, the subject and content of mathematics is sometimes an afterthought at best, or entirely absent, with primary focus put on literacy, history, or science. Is this because classroom mathematics is still seen as the learning of facts and algorithms but not a domain of ideas? Is it because there is still a persistent, perhaps unconscious, belief that some students are wired to do well in mathematics, but others are not?

In the video clips you will be studying, the mathematics students are engaged in centers on the core curriculum content of basic operations—addition, subtraction, multiplication, and division—but it is about formulating and investigating generalizations rather than explicitly learning strategies for solving individual arithmetic problems. The development of accuracy, flexibility, and fluency in solving arithmetic problems is, in itself, an important and fascinating topic. What is also important is for young students to dig even more deeply into the basic operations, through three practices that are highlighted in these videos: noticing patterns and regularities in the number system, conjecturing about what is general in those patterns, and creating a variety of representations to show why and how those patterns hold.

For example, when first graders encounter the operation of addition, they naturally begin to notice and describe patterns. Imagine a group of students generating addition combinations that make 10. A student says: *4 plus 6 makes 10, so 6 plus 4 has to make 10*. Another student chimes in: *I have another* *one—8 plus 2 is 10, so 2 plus 8 works, too.* And a third student says, *but 5 plus 5 is in the middle so it doesn't have a turn-around.* These students, at the beginning of their mathematical journey into addition, are already starting to notice something that is true about addition *in general*, and they have invented a name, "turn-arounds," for their idea (what they will, in later years, encounter as the commutative property of addition). What if the teacher were to challenge them further to state precisely what they mean by "turn-arounds"? What if the teacher were to ask, *Does this work just for these numbers, or does it work for other numbers, too? How do you know?*

In this book, you will see young students investigate patterns in the number system with intelligence and enthusiasm. When given the opportunity by teachers who show genuine interest in their ideas, students enact their intellectual agency by bringing to these problems their own ways of thinking, and they express their identities as they develop and explain their own models, pictures, and diagrams. The teachers in this book assume the brilliance of their students (Aguirre et al., 2024; Delpit, 2012; Leonard & Martin, 2013; Lewis, 2018; National Council of Supervisors of Mathematics & TODOS, 2021) and support them to develop their intellectual power.

The mathematics content in this book, then, is about the core idea of *generalizing*: finding and proving what holds true across multiple, related examples. The generalizations explored by students in the videos are fundamental to a complete and deep understanding of the operations and connect elementary arithmetic to later study of algebra. Just as important, the mathematical practices that are part of generalizing—noticing patterns, articulating conjectures, representing how and why a general pattern holds—create a fertile context for the development of collective mathematical agency. It is content that has many entry points and can be accessed through multiple modes of participation. You can find a summary of the generalizations that students work on in the examples in this book in Appendix A. Teachers in the video were working from lesson sequences which we wrote in collaboration with another group of teachers. Interested readers can find the full lesson sequences in the book *But Why Does It Work?* (Russell et al., 2017).

Finally, we want to make clear that the mathematics content of these lessons is only one aspect of students' mathematical study—a deep dive into the structure of numbers and operations. It is not the only kind of mathematics investigation students should encounter. We advocate a rich mixture in students' mathematics curriculum that includes becoming fluent with a variety of calculation strategies, constructing and analyzing geometric objects, collecting and describing data, studying how people from different backgrounds and cultures have used mathematics, and undertaking projects to answer questions about the world using mathematics (National Council of Supervisors of Mathematics & TODOS, 2016). Important work is being done, including by contributors to this book, to create investigations in which students use mathematics to interrogate issues coming up in their own communities (e.g., Aguirre et al., 2019; Cirillo et al., 2016; see also Appendix B). All of these experiences help to raise students' voices with an emphasis on making sense, being curious, asking questions, and taking risks.

In this book, we are exploring the ways in which teachers support every voice in their classroom to engage with ideas about numbers and operations. These investigations are not limited to calculation but, rather, challenge and inspire students to delve into underlying mathematical structures.

How to Use This Book

Each chapter of this book focuses on an aspect of building a community that weaves deep mathematics with equitable participation and raises questions for reflecting on practice. The following suggestions will help users of this book to get the most from reading and viewing.

Do the Math

Because the mathematics of noticing, conjecturing, and representing general ideas about the operations may be unfamiliar, this resource offers readers an opportunity to investigate some mathematics for themselves as a way to introduce the concepts that students in the video are working on. We strongly suggest engaging with these investigations before watching the related videos in order to get a sense of the complexity of the ideas with which students are working and to better interpret what the students are saying and doing. (If you are like us, you may enjoy adding some math to your day!)

Watch the Classroom Video

One to three two- to eight-minute classroom video clips are the focus of each chapter. We can't say this strongly enough: *Watch the video*. It is possible to understand some of what is going on in the classroom by reading the text and transcript, but seeing the students—noticing their gestures and expressions, hearing the confidence or hesitation in their voices, waiting out the silences—are aspects that don't come through on the page.

When you watch a classroom video, it's tempting to quickly take on a stance of criticism, looking for what the teacher "should have done." Keep in mind that none of us viewing these short clips have the teacher's knowledge about their students or the context of the lesson—what happened before or what the teacher intends to do next. Viewing the video is an exercise in close observation: What do you notice about what students say and do? What does that imply about student learning? What do you notice about what the teacher says and does, and what do students say and do in response?

In most chapters, we recommend that you view each video clip twice, using different lenses. The first time through, focus on what students are learning and the way the teacher gives students access to deep mathematics. What are the ideas students are coming up with? How does the teacher respond to these ideas? In the second viewing, think about students' participation. What opportunities and supports enable different students to dig into the mathematics, express ideas and questions, and interact with other students and with the teacher in building ideas? Is there evidence that students are developing identity and agency as math learners? We provide specific Reflection Questions for these two viewings for each clip.

Transcripts for videos appear at the end of chapters for your reference as you reflect on and discuss with colleagues what you have seen. These are not intended to replace watching the video, since the transcript does not capture gestures, facial expressions, the length of pauses, or other factors that may be important in considering what is happening in the class. Further, while the audio of the video is generally good, and one of us was always making running notes of what students said while the other videotaped, it was not always possible to be certain of students' words. In these cases, we did our best to transcribe accurately, but there may be mistakes, and in a few cases, we have written "unintelligible" to indicate that we could not make out what a student said. There are three aspects of speech that you will hear in the video that we did not capture in the transcriptions: (1) some repetitions of words or phrases made by the teacher or student (e.g., both teachers and students often start a sentence, then restart it); (2) nonword interjections, such as "um"; and (3) variations of pronunciation (e.g., for "going to," people often say "gonna" or variations in between the two). The transcripts, then, are aids for remembering what you heard and saw in the video, but they are not substitutes for watching the video.

Read and Reflect on What Others See in the Video

We believe that we learn best in community—a community in which our tentative thoughts are welcome, in which we try to truly hear and understand others' ideas, in which we challenge each other and ourselves to think about our beliefs and actions, and in which we build stronger commitments than we might be able to sustain on our own. In that spirit, we invited the teachers in the videos and also a small number of educators, representing a variety of backgrounds and roles, to provide brief commentaries on the video clips. The six Collaborating Teachers and seven Critical Friends (introduced in the Preface) all bring to this work both a deep interest in mathematics teaching and learning and a focus on lifting up student voices, especially the voices of students from groups who have been historically marginalized. There is no simple list of strategies that, if implemented, will establish a dynamic mathematics community that includes every student. Rather, teachers' *ongoing, persistent, and determined reflection* on their classroom practice and its effects on student learning is the critical factor.

The purpose of the commentaries in each chapter is to open up different perspectives for viewing each video lesson. As the commentators viewed these classroom videos, they made different observations, raised different questions, and took away different ideas to apply to practice. Thinking about the reflection questions that follow each commentary can help you, alone or with colleagues, choose themes to focus on. Some of the commentaries will undoubtedly connect with issues you have already identified in your practice, while others may provide unexpected questions about aspects of teaching, learning, and participation that have been less visible to you.

In our own years of teaching and of collaborating with teachers over many decades, we, the authors, have consistently found that collaborating to reflect on practice is powerful. Teachers can and do raise questions about their own practice alone, but considering the observations and questions of other educators ensures that one's own questions are not restricted to established routines and beliefs. For that reason, we encourage you to find a partner or form a study group to reflect together on what can be learned from the students, teachers, and other educators who have contributed to this book.

Take Next Steps

Each chapter concludes with suggested "next steps" for you to try in your own practice. We have written these suggestions with the hope that you will explore new ways to ensure equitable participation of your students while maintaining your commitment to deep mathematics.

Notes Organizer

The Notes Organizer is an electronic supplement found at companion.corwin.com/courses/equitable-deep-math. It provides the math activity, Reflection Questions, and Next Steps from each chapter, with space to take your own notes.

Facilitator's Guide

The Facilitator's Guide (also available at companion.corwin.com/courses/equitable-deep-math) is designed for those who lead professional development or a study group based on this book. The guide suggests a chapter-by-chapter plan for organizing study group meetings and offers tips for facilitators.

A Note About Some Terms We Use

There are some terms common in the education community that can refer to different things. To avoid confusion, we specify here what we mean when we use the terms *mathematical practices* and *mathematical representations*.

By *mathematical practices*, we mean the working practices of professional mathematicians. A number of different documents, such as the National Council of Teachers of Mathematics *Principles and Standards* (NCTM, 2000) and the *Common Core Standards* (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), have offered lists of practices. Our use of the term includes these but is not limited to them. For example, two key mathematical practices illustrated in this book are *noticing patterns* and *formulating conjectures.*

Mathematical representations are physical, visual, or verbal depictions that embody a mathematical object. Mathematical representations may include pictures, diagrams, number lines, graphs, arrangements of physical objects, mathematical expressions, equations, or the statement of a generalization. Because story contexts can carry so much meaning about mathematical relationships at the elementary level, we include them as representations as well. *Student-created mathematical representations* are drawings, diagrams, models, story contexts, and so on, that come from students' imaginations, as well as more standard forms of representation, such as number lines or arrays, that students have incorporated into their repertoire.

Reflection Question

This resource is structured around four aspects of mathematics community: *every voice matters; collaboration supports student agency; student-created representations offer anchors, openings, and depth;* and *students are initiators and advocates for their own learning.* Think of a recent experience in your own context. How were each of these aspects present? If you were to choose one on which to focus right now in your own work, which would you choose and why?

Part One

Every Voice Matters

First-grade teacher Isabel Schooler describes different ways students have "voice" in her class: They can ask questions, offer what they notice, draw or build clear representations, restate others' ideas, ask for clarification, agree or disagree, suggest other ways, give a partial answer, or provide mathematical language.

All of these modes are ways for students to enter the conversation, to make their ideas known, and to be listened to and appreciated for their thinking about mathematics. "Voice" is not always oral. As Ms. Schooler suggests, students can contribute to discussion by drawing or building a representation to show their thoughts. They might also use signals, like the hand motions you'll see in some of the videos that indicate "I agree" or "I'd like to build on that idea." Even a simple gesture, like pointing to a part of a representation, may allow a student to give voice to an idea.

The chapters in Part One focus on how teachers support each student to develop and use their voice in mathematics.

The following are the major themes of Part One:

- A mathematics community that is focused on deep mathematics and equitable participation allows all students to develop agency as mathematics learners.
- Discussion that is focused on deep mathematics involves a nexus of ideas, offering students different entry points and different modes of participation.
    - To facilitate such discussions, teachers must develop a set of skills and dispositions that promote student engagement.
    - To participate in such discussions, students must also develop a set of skills and dispositions.

In Part One, through viewing videos, reading what our Collaborating Teachers have written, and considering ideas and questions posed by our Critical Friends, we'll delve into the complicated work that teachers do to listen to and sustain every student's voice, while weaving all of those voices into the development of rigorous mathematics for the whole class.

Chapter 1

Creating Multiple Openings Into Engaging Mathematics

Our work is based on two assumptions:

1. Mathematics is an interwoven network of ideas.
2. Students come to school with mathematical ideas, and part of the work of the teacher is to draw out those ideas and help students develop them further.

Students in the elementary grades are learning not only mathematics content but also what mathematics is and what it means to be mathematical thinkers. We view the learning of mathematics as an active endeavor—the construction of ideas, rather than passive absorption of delivered knowledge. As students find their place in the mathematics classroom, we want them to learn that they are capable of making sense of complex mathematical ideas. We want mathematics classrooms to be settings where students have the opportunity to develop positive and productive mathematical identities as doers of mathematics. We hope for mathematics classrooms that are contexts for developing mathematical agency, where students learn to investigate mathematics, share their own ideas, interact with classmates' ideas, and take responsibility for their own learning.

Given this description of what a mathematics classroom can be, we pose, again, the question we asked in the Introduction: How do we ensure that every student contributes a voice to a mathematics community, including students who have been historically marginalized in mathematics, students who have not believed they have mathematical ideas that are important to share, or who, when they have tried to express their ideas, have not been heard?

In this chapter, you will

- do some math for yourself to become familiar with the ideas you'll see students working with,
- watch a video to explore how first-grade students are discovering their own mathematical agency and identities in a whole-class discussion,
- read what our Critical Friends and the classroom teacher have to say about students' learning and participation, and
- consider how to create openings to ensure that every student contributes their voice to the mathematics community.

Whole-class discussion is one of the primary forums in which students are invited to use their voices to contribute mathematical ideas. Take a moment to reflect on your own experiences as a participant in a discussion. Can you remember a time when you have been in a group that seemed closed to your ideas? Did a few people dominate the discussion? Did others seem more certain, more on top of what they had to say? Did you worry that you weren't smart enough or experienced enough to contribute? Or were you usually eager to get out your own ideas? Did you try to understand others' ideas? Was there enough time to hear multiple perspectives?

Now think about what a classroom mathematics discussion might feel like from the perspective of an elementary-school student. Is it a setting in which students are willing to put out tentative ideas, clarify them, ask questions, and build mathematics together? Might it be intimidating for some students? What does it take to create a classroom community that works on deep mathematics and also supports students in participating with all their different personalities, backgrounds, facility with language, and mathematical understanding?

As you consider the classroom video and the commentaries in this chapter, we'd like you to think about three aspects of the principle that *every voice matters*. These three aspects of building community lay a strong foundation for interweaving rigorous mathematics and equitable participation.

1. The mathematics content is powerful, engaging, and challenging, but there are many entry points into the ideas.
2. Accessing the depth of the ideas takes time. "Productive lingering" on a few related questions allows students to dig deeply into mathematics concepts.
3. Teachers can create multiple openings into the mathematics. Students can have voice in different ways.

Do the Math

As we explained in the introduction, you'll get more out of the children's thinking in the video if you do some related math work before viewing. Even though you may find the mathematics itself familiar, doing some work of your own with the same mathematical ideas with which the students are working will help you understand its complexity and importance.

1. Make a drawing or diagram or build a physical model for each of these equations:

   $$3 + 6 = 9$$

   $$9 - 3 = 6$$

   $$9 - 6 = 3$$

   Can you draw or build a single representation that shows all three of these equations?

   Explain to a colleague how your representation shows all three equations.
2. The three equations are an example of a big idea about the relationship between addition and subtraction. Can you write an "if . . . , then . . . " sentence that expresses the general relationship among these three equations?

   If . . . , then . . . .

   Can you use your representation (drawing, diagram, or physical model) from Question #1 to explain why your "if . . . , then . . . " sentence is true?
3. Share your "if . . . , then . . ." sentence with a colleague. What is the same about your sentences, and what is different? How do your representations demonstrate why your statements are true?

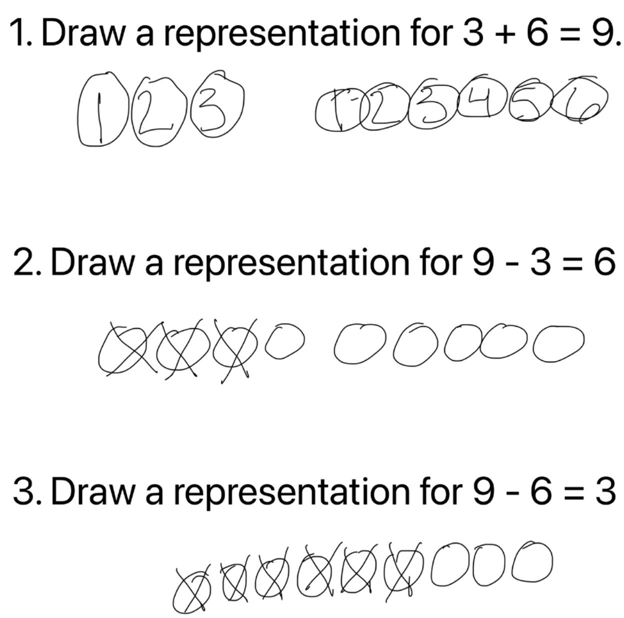
Watch the Video: "Where Do You See the 3?"

To begin to understand what it looks like to create openings for every student, let's watch and analyze Video 1.1. This lesson is from a sequence of lessons in which Natasha Gordon's first graders are investigating the relationship between addition and subtraction, a complicated idea for young children who are new to thinking about these operations. In order to understand the idea, not only must students conceptualize what each operation does, but they must hold in mind images for *both* operations *at the same time* to understand how they are related. At first, students notice this relationship with specific numbers, but then they gradually consider how it applies to all the numbers with which they are working.

In the previous lesson, Ms. Gordon asked students to create a representation for each of the three equations: $3 + 6 = 9$, $9 - 3 = 6$, and $9 - 6 = 3$. The activity sheet also included the question, "What is the same in your representations, and what is different?"

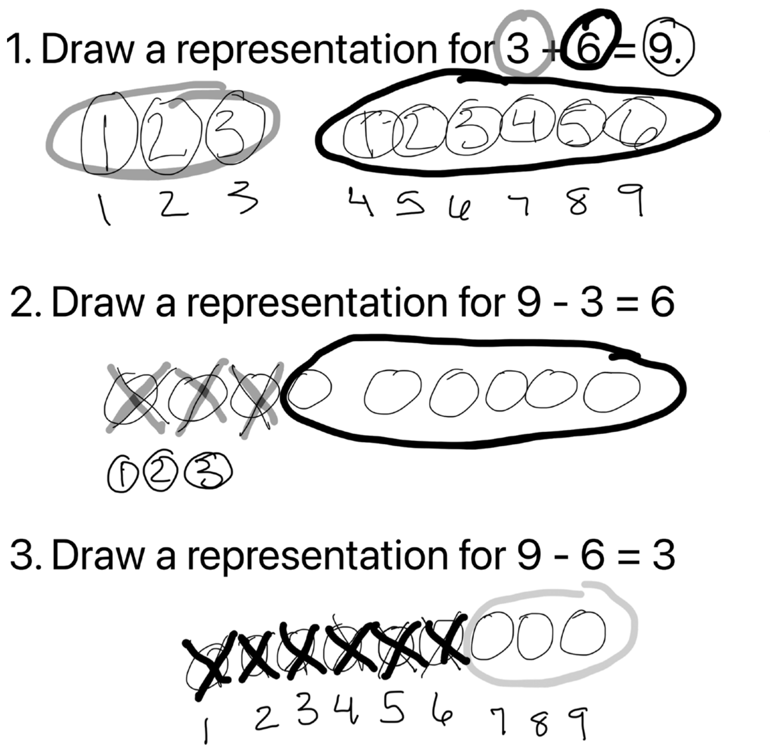
For the lesson shown in the video, Ms. Gordon selected a piece of student work (Figure 1.1) to project onto the whiteboard and discuss with the class.

Figure 1.1 • A First Grader's Work Representing $3 + 6 = 9$, $9 - 3 = 6$, and $9 - 6 = 3$



Before the part of the lesson you're going to view, the class talked about what they noticed is the *same* in the three equations and the three representations. The whole-group discussion was paused for a few minutes while students talked with partners about what they noticed—a classroom structure commonly referred to as turn-and-talk—and then students shared their ideas in the whole group. As they spoke about what they saw as similar in the three representations, Ms. Gordon noted their ideas by using colored markers on the whiteboard. She used one color to indicate groups of 3 in each of the representations (shown with gray in Figure 1.2) and a different color to indicate the groups of 6 (shown in black). By the end of the day's discussion, of which this video clip is a part, the whiteboard looked like Figure 1.2.

Figure 1.2 • Ms. Gordon's record of what students noticed about what is the same in the three equations and representations. She used different colored markers for the groups of 3 and groups of 6, shown here as gray and black, respectively.



During the discussion about similarities, one student, Livia, pointed out that the numbers are in a different order in the three equations. We'll join the group as Ms. Gordon comes back to Livia to ask her to elaborate her idea.

You'll view the 5-minute video clip twice, with different lenses, in order to help you think about weaving together deep mathematics content and openings for students' voices.

First Viewing of the Video: The Mathematics Students Are Working On

Video 1.1

"Where Do You See the 3?"

qrs.ly/fqfs4v8

To read a QR code, you must have a smartphone or tablet with a camera. We recommend that you download a QR code reader app that is made specifically for your phone or tablet brand.

Watch the video clip, "Where Do You See the 3?," with a focus on the mathematics content students are learning.

Reflecting on the Video: The Mathematics Students Are Working On

[You may want to use the transcript at the end of this chapter as you consider these questions.]

1. What are the important mathematical ideas in this clip?
2. How are students engaging with these ideas? What different ideas are students working on? Are there different entry points into the mathematics?
3. In what ways is the mathematics challenging and engaging for the students?
4. What does the teacher do to focus the discussion and to promote persistence with complex ideas?

Second Viewing of the Video: Finding a Way in Through Multiple Modes of Participation

Watch the video clip, “Where Do You See the 3?,” with a focus on how students have voice during this discussion.

Reflecting on the Video: Finding a Way in Through Multiple Modes of Participation

[You may want to use the transcript at the end of this chapter as you consider these questions.]

1. What do you notice about different modes of student participation? Are there opportunities for students to find different openings into the discourse?
2. How does the teacher support students’ voices in this discussion?
3. How does the posted student work and Ms. Gordon’s use of colored markers support students to find ways in to participate in the whole-group discourse?
4. If you were the teacher in this classroom reflecting on this lesson, what might you want to make note of in order to strengthen student participation? Are there aspects of the lesson that worked well to create openings into the mathematics? Are there questions you have about how you could better encourage students’ voices and help students develop agency as mathematicians?

Read and Reflect on What Others See in the Video

Let's return to the three aspects of the principle that *every voice matters* listed at the beginning of this chapter:

1. The mathematics content is powerful, engaging, and challenging, but there are many entry points into the ideas.
2. Accessing the depth of the ideas takes time. "Productive lingering" on a few related questions allows students to dig deeply into mathematics concepts.
3. Teachers can create multiple openings into the mathematics. Students can have voice in different ways.

In this section, you will encounter reactions from some of our Critical Friends to the class session—what they notice and questions they raise. Also included is commentary from Ms. Gordon about how she was thinking about students' entry into the mathematics content during a similar lesson.

1. Critical Friends Consider How These Young Students Encounter the Mathematics Content

"Fact families" are a familiar topic in elementary classrooms. If you search for "fact family" online, you find many worksheets in which children are asked to fill in a series of blank equations with sets of numbers such as 3, 6, and 9. But, without a focus on making sense of these relationships, students might learn to fill in the blanks correctly without thinking about how and why these equations are related.

In Ms. Gordon's lesson, students explore the relationship between addition and subtraction by digging deeply into one example. Young students first need to understand each problem in itself. Many students in first grade are still solidifying the idea that addition can be seen as joining and subtraction as removing, and they are working through what each component of an addition or subtraction equation stands for. Students are also learning what it means to represent a mathematical equation: How do the circles and x's in the drawing represent adding or subtracting? How are the drawings and the equations related to each other? By tracking how each of the three quantities appears in the equations and in the drawings—different ways of seeing how 9 is composed of 3 and 6—students are also taking a step toward understanding more generally how addition and subtraction are related. Because of the many different components of this complex idea, different students might be working on different aspects of the concept within the same lesson.

Here is what some of our Critical Friends have to say about the math in this lesson.

Hetal Patel: The teacher sticks to the math and allows the students to describe it. There's a lot of descriptive language that the kids are using, and she sticks to the words they use. The students have multiple opportunities and a variety of ways to use that descriptive language for what they are trying to make sense of for themselves or for each other.

Virginia Bastable: It feels important to say, Don't wait until you think the idea can be solid for the students. You can help them begin to think about a messy idea. None of the kids seem upset that they're not saying the whole idea. There's nothing negative going on. They're sort of playing with this idea. And even if you're not sure they're going to consolidate it in whatever time period you have, it's still worthwhile. I think that's an important message for people, especially in this day and age when every 20 minutes you have to check off that you met some objective!

Darlene Ratliff: For young students, the mathematics can be murky. Here's how a first-grade class "murks" through it. Nevertheless, the ideas still emerge during the lesson.

Reflecting on the Mathematics

1. Hetal Patel notices how the teacher makes use of students' own words. Refer to the video or the transcript. How does the teacher encourage and accept students' own language? How does she help students expand and clarify their language, both for themselves and for the understanding of other students?
2. What is your response to our Critical Friends' thoughts about engaging young students with challenging mathematics that is "messy" or "murky" for them or that you may not be able to bring to closure? What do these observations have to do with students developing their identities as doers of mathematics?

2. The Teacher Reflects on a Similar Lesson: What Are Students Learning?

The following year, Ms. Gordon taught the same lessons in her first-grade class. Students represented and discussed the three equations involving 3, 6, and 9 in a lesson similar to the one you watched on video. Based on what she observed in that lesson, she planned the next lesson to focus on two related story problems, using the numbers 6, 9, and 15. This is an example of "productive lingering"—investigating a small set of related problems thoroughly in order to dig into significant mathematics, in this case, the relationship between addition and subtraction. Rather than practicing with pages of sets of problems similar to $3 + 6 = 9$, $9 - 6 = 3$, and $9 - 3 = 6$, students have the opportunity to examine another single example—allowing for time to draw, discuss, and ask questions. The ideas are developed across several lessons. Afterward, Ms. Gordon reflected on the students' representations and discussion of the story problems.

Natasha Gordon: In the session about 3, 6, and 9, students stated that the subtraction equation MUST have a specific answer because of the similarities in numbers in the addition equation. I wondered whether they understood what was happening with the numbers as they related to the actions of addition and subtraction. Going into the next session, I was curious as to whether students would be able to better articulate their understandings and connections, as well as if any new ones would emerge.

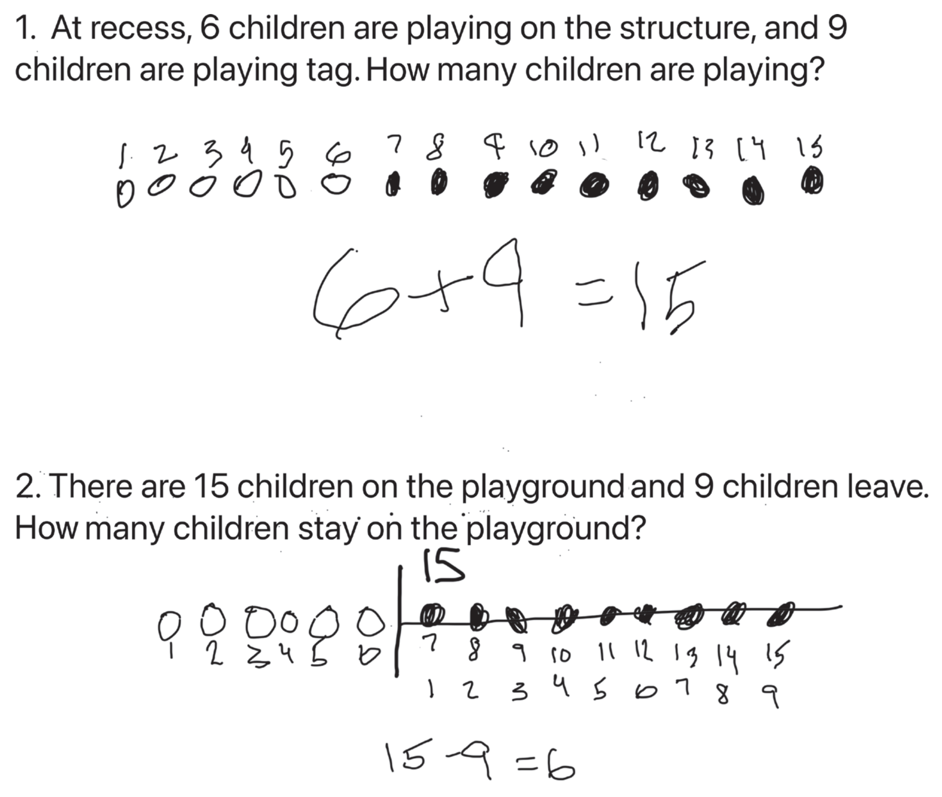
For this session, students drew representations for the following problems:

1. *At recess, 6 children are playing on the structure, and 9 children are playing tag. How many children are playing?*
2. *There are 15 children on the playground, 9 children leave. How many children stay on the playground?*

Students were asked whether the first problem helped them solve the second problem and, if so, how?

When we came together to analyze one student's work (Figure 1.3), the discussion was filled with many student voices as we tried to make sense of each problem and each representation. When I asked whether the first problem could help you solve the second problem, I heard the following comments.

Figure 1.3 • A Student's Work Illustrating Two Related Story Problems



Reynald: Yes, because 6 + 9 = 15 and 15 – 6 = 9, because if you have 15 and you take away 9, it will equal 6 because 6 + 9 = 15.

Ms. Gordon: Can someone add onto Reynald? How is that helpful if you know 6 + 9 = 15? He said then 15 – 9 will equal 6. How is that helpful?

Xavier: Both problems, both have 15, but on the bottom one of the problems, the other 9 was taken away, and the first one they were still there, so they went somewhere else.

Ms. Gordon: Who can say more? Xavier pointed out that there were 15 children in both problems. We can see the 15 circles in both problems, and also I see 15 in both equations. But you said in one problem the 9 is taken away? And then in the other problem what happened with the 9?

Xavier: The 9 are still playing.

Ms. Gordon: OK. So in one problem the 9 are taken away, and in the other problem the 9 are still there. Someone else say more . . .

Tatianna: I was talking about this with my talk partner because it *does* make sense because this one is starting with $6 + 9$ and it equals 15, and if it's 15 and it takes away 9, it will leave back where we started with 6.

Ms. Gordon: Interesting. Something about this problem [points at top problem and underlines the 6 in the equation $6 + 9 = 15$] we *started* with 6. And you said this problem [points at bottom problem and underlines the 6 in the equation $15 - 9 = 6$] we're *left* with 6. So how is that helpful? Someone, add on even more. Thank you, Tatianna. This problem we started with 6, and we ended with 15 children altogether. And with this problem, we started with 15 children, and we're left back with 6, Tatianna said. Miriam, do you want to add on?

Miriam: I just want to say it's like you're putting the stories together because if 15 children were playing on the playground, and 6 were playing on the structure and 9 were playing tag, you can put both stories together by, after a little bit of them playing, 9 children left.

Ms. Gordon: But can you say more? How can you put those two together? So you said after some time . . .

Miriam: It's like, it's basically like 9 children come in on the playground and then those 9 leave.

In this five-minute dialog, four students built on each other's ideas and collectively found a connection between the two problems: that the nine students who came onto the playground to play tag could be the same nine students that left the playground, leaving us with the original six children. Tatianna stated that "it does make sense" as she added on to her peers' ideas and tried to explain the connection further. Her exclamation was a sign of her bringing the math together and tying it into the context of the problem. I do not believe all students have arrived at this understanding, but I do believe that as this relationship is explored further, more students will approach this understanding.

Reflecting on Ms. Gordon's Writing

1. As Renald, Xavier, Tatianna, and Miriam discuss the problems about children on the playground, how do their comments build on one another? What are they saying about the relationship between the two problems?
2. What is your response to Ms. Gordon's last sentence in her comments, "I do not believe all students have arrived at this understanding, but I do believe that as this relationship is explored further, more students will approach this understanding"?

3. Critical Friends Consider How Students Are Offered Multiple Modes of Participation

A teacher's responsibility to promote equitable participation is complicated. When using video clips in this book to reflect on our own practice, we are seeing a very small slice of classroom activity. We do not know what the teacher knows about what came before what we see, how the teacher is supporting the needs and strengths of individual students, or how the teacher plans to follow up. Some aspects of what happens on the video are invisible to us. For example, when Ms. Gordon later reflected on the lesson, she commented, "When we broke into turn-and-talk, I made a point of visiting pairs of students who had not yet spoken up. I noticed that four of those students subsequently contributed to the whole-group discussion."

We use these brief video clips not to comment on what the teacher "should have done," but as learning tools for ourselves—to reflect on what we see, to notice which teacher moves bring students into the conversation, to ask questions and wonder what we might do to ensure equitable participation in our own contexts. Even without knowing all that the teacher knows, we can use these short excerpts to provoke questions for our own practice, such as these that our Critical Friends and Ms. Gordon's second-grade colleague, Quayisha Clarke, raise. Note that they viewed the full 30-minute lesson from which this clip is taken.

Quayisha Clarke: Within the turn-and-talk and within the discussion, Natasha [Gordon] would ask questions like, "So what do you think about Livia's idea?" It's really evident to the students that their ideas matter because the teacher is listening to them, and so are the students. What everyone says matters because they're going to talk about this new thing that just happened. I think that gives a lot of power to student voices. I counted, too, and I think it was almost everyone except two students who talked in the whole-group discussion. That doesn't just happen. I'm not going to just walk into any classroom and see all of the students talk in a 30-minute math discussion. She's given access to everyone's voice, and power, as well.

Virginia Bastable: When Ms. Gordon does call on people, she gives the students a lot of time to talk. Even if they're stumbling or not saying their ideas well, there's clearly time for them to get through that, and sometimes they do and fix it and sometimes they say, come back to me, I can't finish. I thought that was supportive of increasing voice.

Darlene Ratliff: As one often sees in first-grade classrooms, boys fidget and move all over the place in this lesson. But they're still engaged. I want to emphasize that they're still engaged, even though they are moving around and fidgety. The teacher didn't spend all of her time saying, "Come on, sit down, stop rocking, stop playing." It was all about the mathematics and the discussion, and not about who's fidgeting. Whether they're fidgeting or not, if they're still tuning in to what you need to have happen, that's an important place for those students.

I also want to say that sometimes voice isn't talking. Sometimes, voice is signaling. As we watched this class, I can pretty much say all of them displayed voice in some way, whether in the turn-and-talk or gestures, everyone had some voice. Some had more voice than others, but overall, there was an atmosphere of voice.

Lynne Godfrey: I was looking at the different kinds of questions the teacher posed. It's one thing to go up to the board to point. But I'm wondering, who gets to answer the thinking questions?

Cindy Ballenger: Josiah was so clear and articulate. It was my feeling that he was helping everybody. So then there's the question, Does the teacher always use Josiah in this way, or are there other kids who take on this role? There might be moments when Josiah is the guy the teacher needs to go to, but you have to take a long view.

Yi Law Chan: I noticed a core of eager hands, the very eager waving to signal something they very much wanted to share. I'm also thinking about how the role of those kinds of signals plays out in a classroom that also has other kinds of signals, as well as opportunities for students to do the turn-and-talks. How will these different nonverbal means of communicating impact different students? What I noticed is that there are definitely students along the periphery, physically and also in their involvement in verbalizing, in the whole group. The turn-and-talk allowed every pair of students to verbalize thinking, but I did notice that there's a central group of students who had the floor more often in the whole-group conversation. So I have curiosity around the voice there in the whole-group setting. How do we assess the impact of a student's contribution on their own learning or the impact of others' comments on their learning? It's a question I have that I'd like teachers to think about. When I visit a classroom, I might not be able to assess this impact, but teachers in the classroom have the ability to assess that over time.

Virginia Bastable: Paying attention over time is such an important theme for a teacher. In any one clip, we're seeing so little that it's hard to make these judgments. But as a teacher in a classroom, I need to notice what kinds of questions I am asking and to whom and make sure those thinking questions get more distributed.

Reflection Questions

1. What do you notice in the clip from Ms. Gordon's class about how students have voice? Are different modes of participation evident? Is it important for every student to speak in whole group? How else might students productively participate in the whole-class discussion? What do you think, in your own context, about Darlene Ratliff's statement that "Sometimes voice isn't talking"?
2. While many students have the opportunity to participate in different ways, in this whole-group discussion, there may still be students who do not find an opening to participate, students who, as Yi Law Chan mentions, are on the "periphery." You may have noticed students in the video—and it is likely this would be true in any classroom—who don't seem to actively participate. What would you want to know about these students? How might you follow up with them? What might you plan in order to provide openings for them to engage in the next whole-group discussion?

What Do You Want to Remember From This Chapter?

Take a few minutes to note for yourself ideas you want to hold onto as you continue to investigate the meaning of a mathematics community and how to build it. What teacher moves have you noticed in this chapter that you want to bring into your own practice? Here are some of the ways we like to think about what the teachers and Critical Friends in this chapter have said about creating openings for every student:

- **Create multiple entry points and openings.** A large part of equitable mathematics teaching is access. Give students openings to enter into mathematical ideas through a variety of representations, including words, equations, and diagrams, to ensure the greatest possible participation.
- **Create an expectation of "productive lingering" on important ideas.** Structure lessons to focus on one or a few related questions. Encourage students to investigate those questions deeply, welcoming questions and ideas from many voices.
- **Celebrate different forms of participation in class discussion.** Participation in math discussion can include stating what you notice, asking questions, gesturing, building on classmates' ideas, and indicating agreement, disagreement, or confusion. Make openings for, acknowledge, and celebrate all of these forms.
- **Pay attention over time.** Develop ways to track which students are participating and in what ways. Who speaks up in whole-group discussions? What kinds of questions are you asking to which students? Who is sharing their work? Who is commenting on other students' ideas or representations?

Taking a Next Step

List the students in your class and place their names in one of three groups: (1) one-third of the students who participate most in math class, (2) one-third of the students who participate least in math class, and (3) those who fall between the two. As you look over the names in the three groups, what do you notice? What questions do your lists raise for you? If you don't have your own classroom, adapt this activity to another group with which you are working.

Video 1.1 Transcript: **"Where Do You See the 3?"**

Ms. Gordon: Livia, I'm going to come back to you because you started talking a little bit about how they're different. Can you say more about how they're different? What were you saying?

Livia: So what I was saying was that they're the same numbers but they're switched up, like in the first equation it says 3 plus 6 equals 9. And then the next equation is 9 minus 3 equals 6. And then the next is 9 minus 6 equals 3.

Ms. Gordon: How many friends also see that, that they're the same numbers but they're a little bit different because they're in different orders? For example, Caitlyn started talking about the number 3. Where do you see, in the first representation, where do you see 3? Audrey?

Audrey: The first 3 in the first one.

Ms. Gordon: The first three circles. But in the second representation, where do you see the 3? J'aimeson?

J'aimeson: [Points to board.]

Ms. Gordon: So, these three right here. Audrey, can you say a little bit more? What did you just say?

Audrey: I said the ones that were taken away.

Ms. Gordon: So how is that different? Just in those first two. For the first problem we noticed that the 3 is represented by the first three circles. But in the second problem the 3 is represented by the three that are being taken away. Josiah?

Josiah: Because that one is adding 3, and that one is taking away 3.

Ms. Gordon: How can you tell that this one is taking away 3?

Tierra: Because it says it's taking away 3.

Ms. Gordon: Where?

Tierra: Right here.

Ms. Gordon: Ah, in the equation. 9 take away, 9 minus 3. How is that represented in the work? What did this scholar do to represent 9 take away 3 or 9 minus 3?

Anu: They x'd it.

Ms. Gordon: Ah, they crossed them out. Okay, so we talked about 3, we noticed 3 is represented by the first three circles in the first equation; 3 in the second equation is represented by the three that are taken away. What about the third representation? We said we saw 3 there, 3 is in this equation and in the representation, but where is the 3 in this representation? Tunmiche? What does the 3 represent in the third equation?

Tunmiche: Right there.

Ms. Gordon: Where?

Tunmiche: [points to board]

Ms. Gordon: Can someone say more? Okay, so we see the three are right here. Can someone say more? Josiah, I see that you want to add on.

Josiah: The 3 is represating for what it equals.

Ms. Gordon: OK, what it equals or . . . ?

Anu: The answer.

Ms. Gordon: Who can say more—what it equals, the answer . . . Audrey?

Audrey: Like what it means.

Ms. Gordon: Josiah?

Josiah: The total.

Ms. Gordon: Okay? Interesting, you're saying the total. When we say total, the total is how much we have . . .

Students: Altogether.

Ms. Gordon: Is the 3 what we have altogether?

Students: No

Ms. Gordon: So what is this 3? Janiya?

Janiya: What we take away?

Ms. Gordon: Okay, so we know this problem is subtraction. We are taking away, but what does this 3 represent?

Anu: What remains?

Ms. Gordon: So, in the first problem the 3 is what we started with. In the second problem we took away 3. And in the last problem 3 is what we're left with.